VOLUME 106 PART A NUMBER 28 AUGUST 1959

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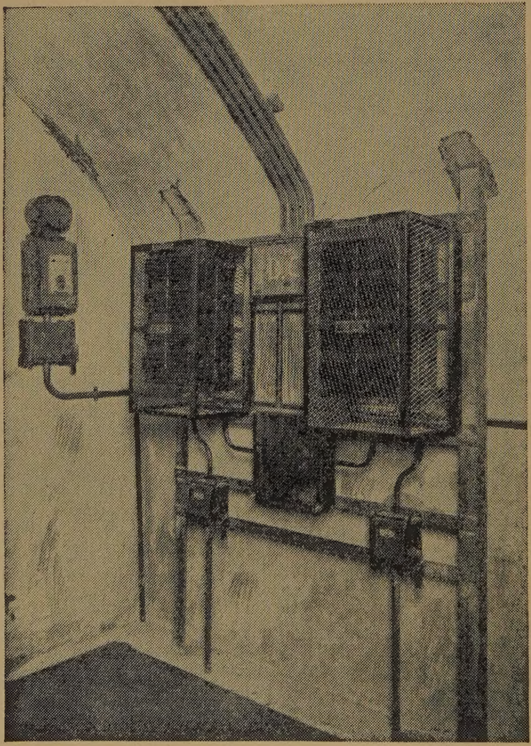
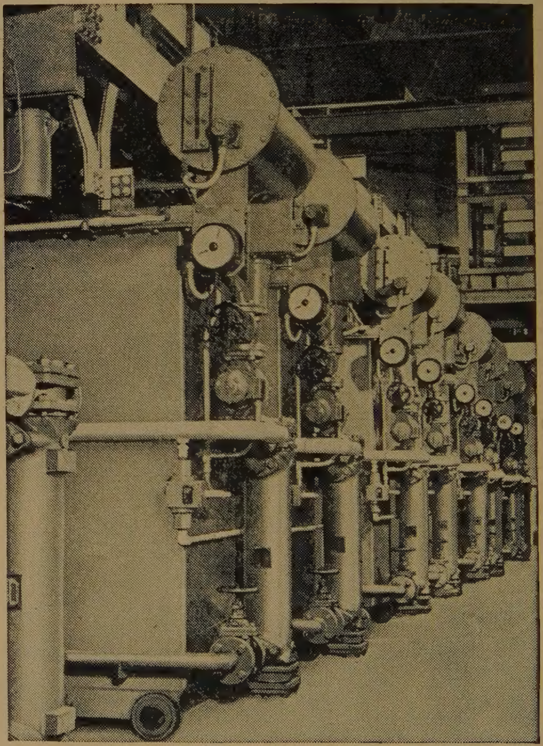


1932 *Two copper-oxide rectifiers supplying power to variable speed motors at the Bruton Club.*



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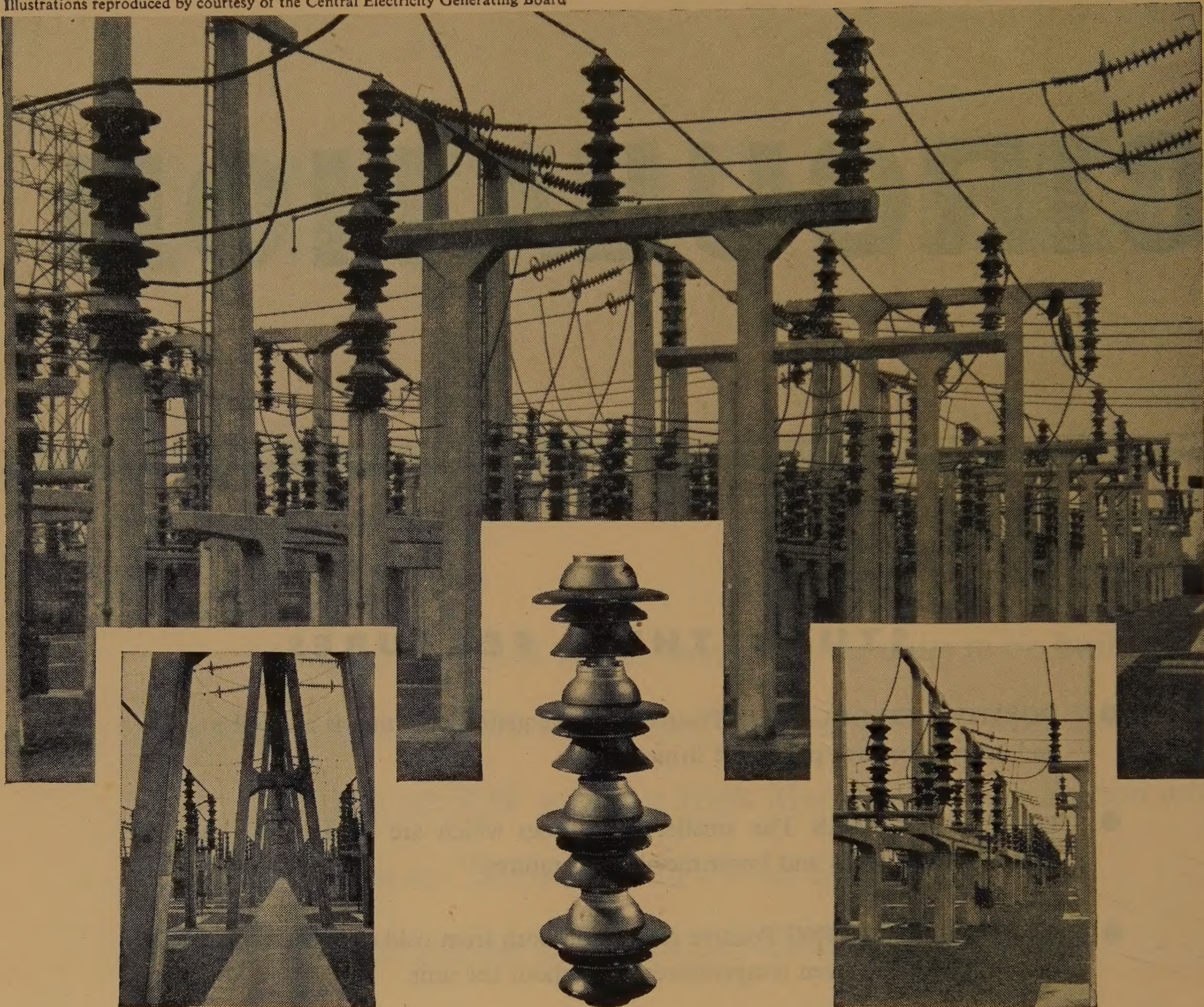

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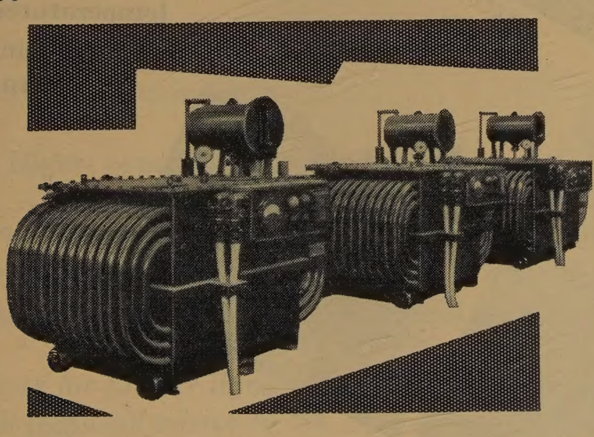

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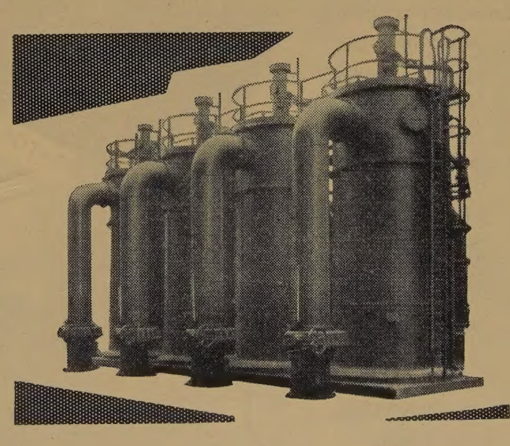

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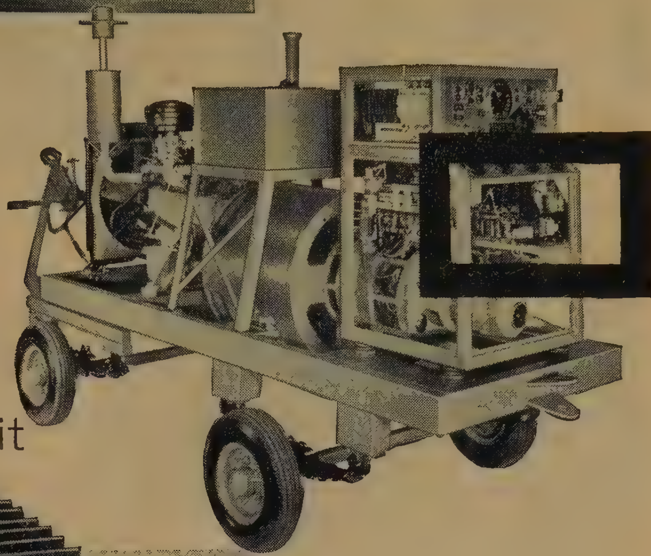

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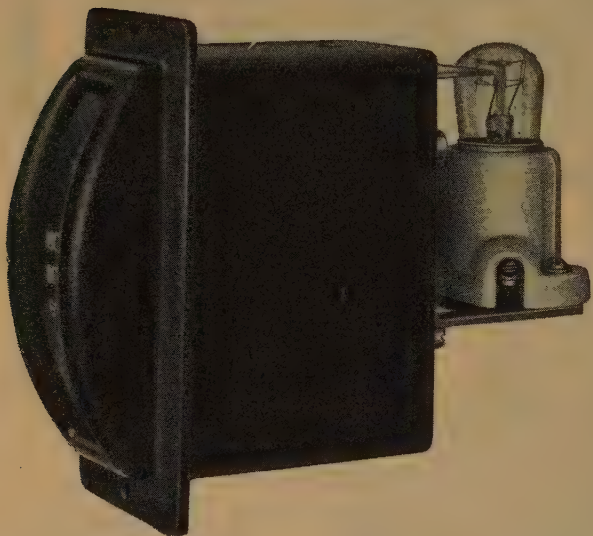

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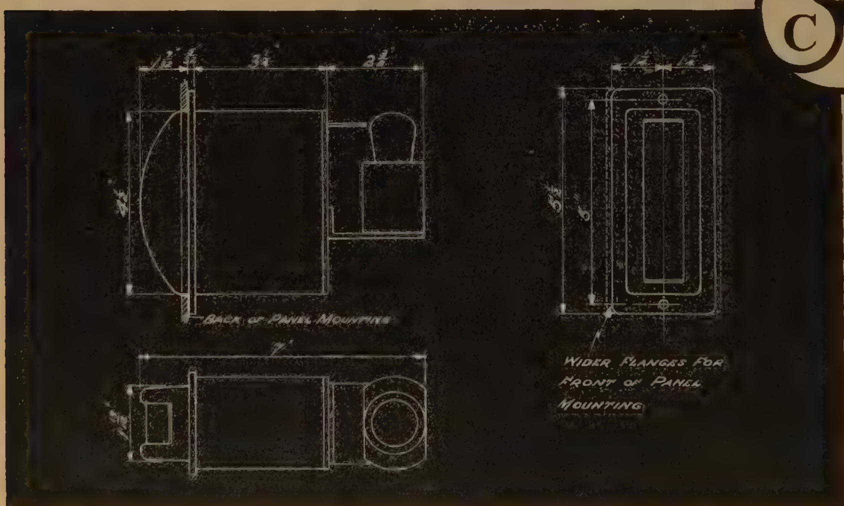

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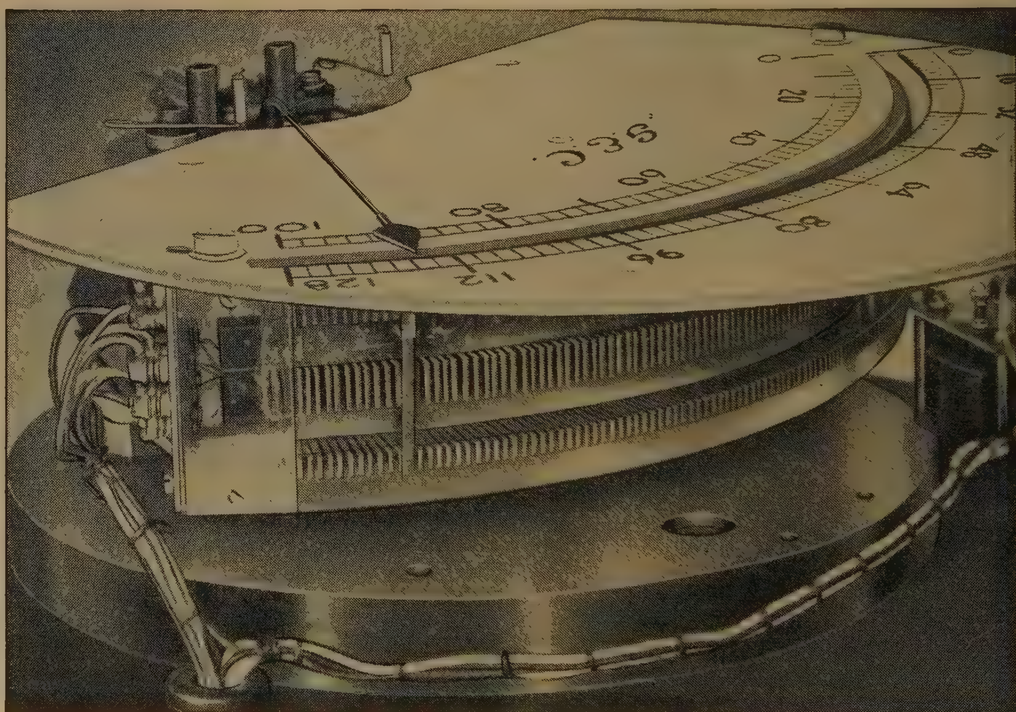

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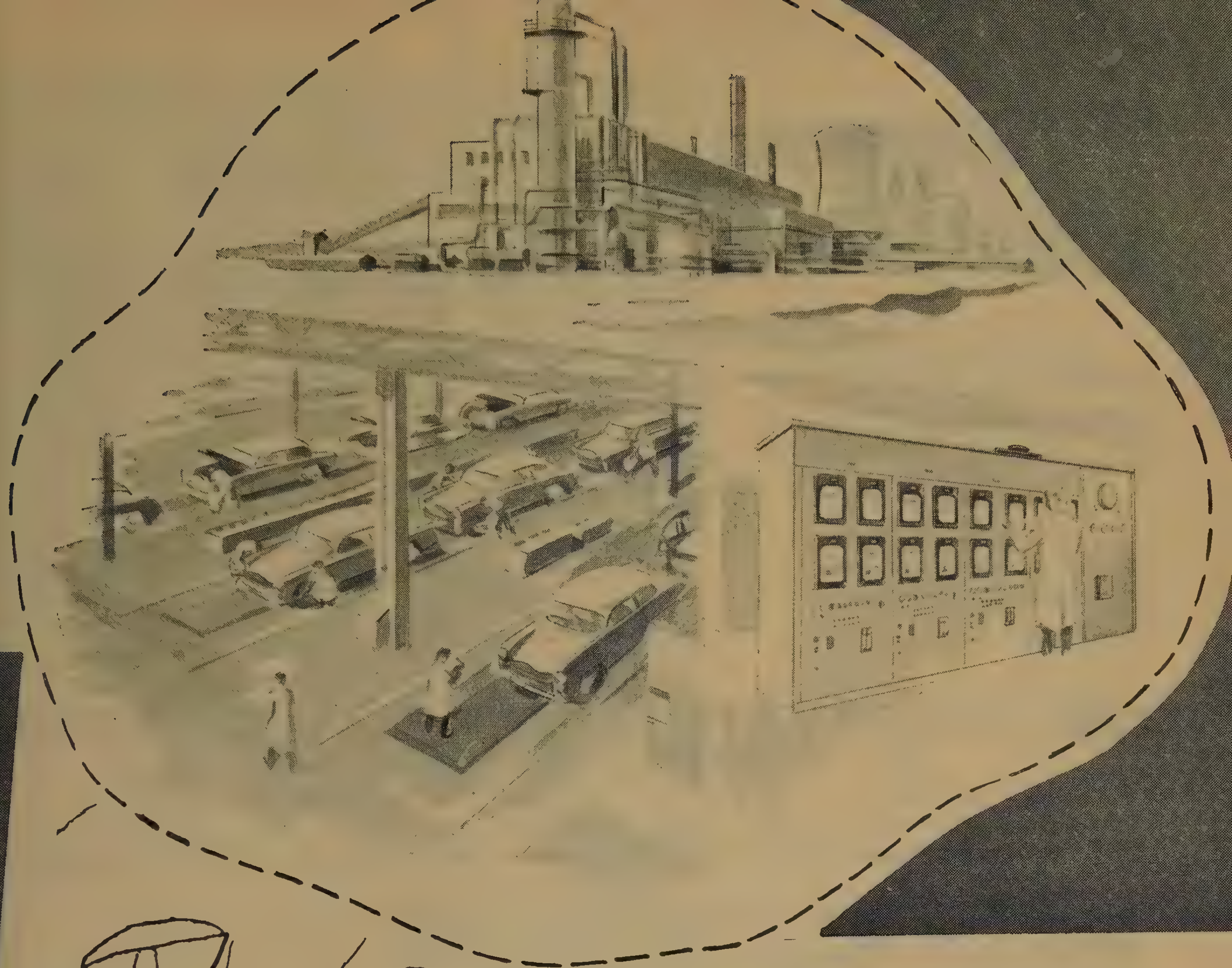

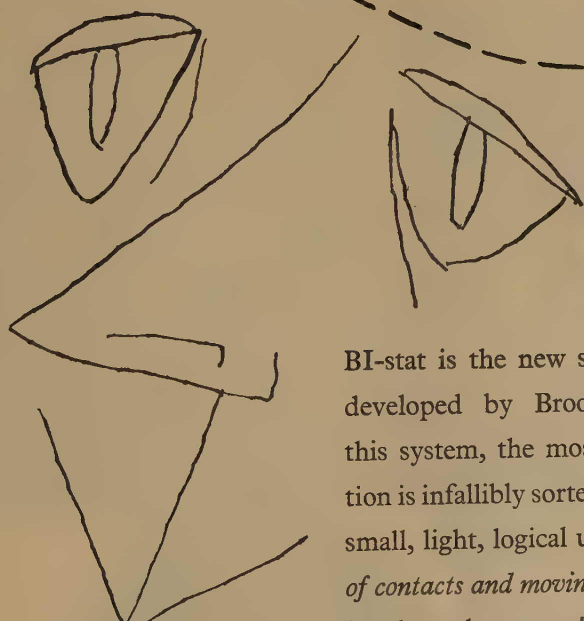

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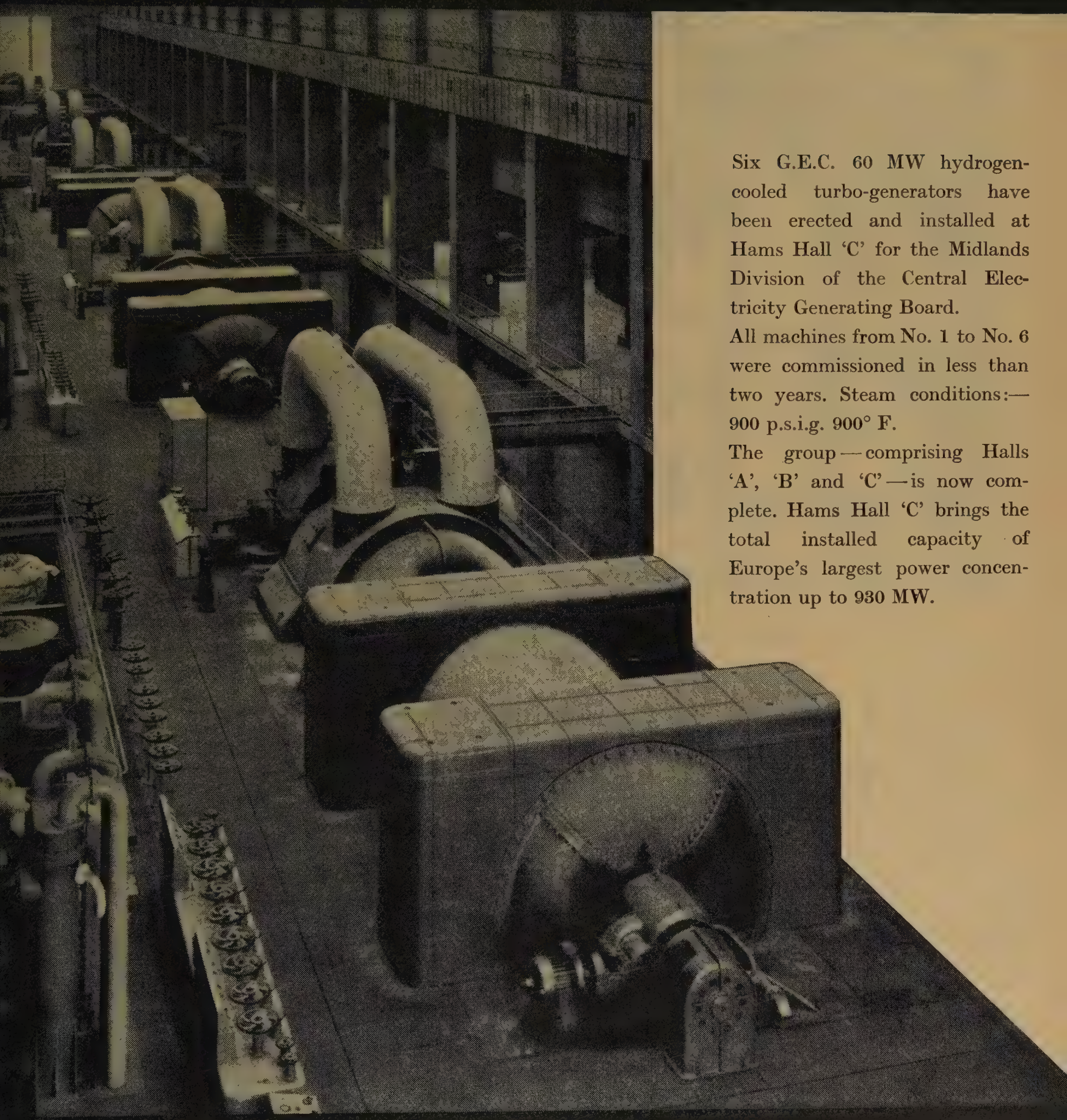

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EDITED UNDER THE SUPERINTENDENCE OF W. K. BRASHER, C.B.E., M.A., M.I.E.E., SECRETARY

VOL. 106. PART A. No. 28. AUGUST 1959

621.317.79 : 621.311.161.017 : 338.58

The Institution of Electrical Engineers
Paper No. 2890 S
Aug. 1959
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AN ANALOGUE COMPUTER TO EVALUATE THE COST OF CHANGES IN POWER ON A POWER SYSTEM

By K. W. WARDROP, B.Sc.(Eng.), Associate Member.

(*The paper was first received* 17*th March, in revised form* 9*th December*, 1958, *and in final form* 29*th January*, 1959.)

SUMMARY

Changes in the cost of supplying power due to changes in total power demand, or due to changes in the allocation of power between generating stations, are evaluated by an analogue computer described in the paper.

Changes in the power lost in transmission are included in the evaluation, and systematic changes in power conditions are made automatically to discover the minimum cost of supplying power.

(1) INTRODUCTION

When power stations are interconnected it becomes essential to specify their power outputs for each hour of the day. The aim of this allocation of power is to ensure that the possibility of supply failure is minimized, equipment is not overloaded, system voltages are kept within specified limits, and, subject to these requirements, the total cost of supplying power is kept to a minimum. The cost of producing power at a generating station varies with the power output of each machine, the price of fuel delivered to the station and many other factors. It is present British practice to assume a single value (sometimes two values) of cost per unit of power for each machine, and to load the machines in order of ascending cost, consistent with the limitations of the power system. The power loss in the system is either ignored or assumed to be a fixed percentage of the transmitted power.

It is changes of power demand and allocation which have to be considered during the hour-to-hour operation of a power system. In making comparisons, only costs which are not common to all alternatives need be considered, and so only the costs due to power changes have to be evaluated to determine the minimum total cost of supplying power. Furthermore, the cost of the change in transmission loss may be a significant part of the cost of such a power change. The cost of any change or redistribution of power, ignoring the transmission loss, is simply assessed from the generating costs at each power station. The change in transmission loss is the sum of the changes of power loss in the various transmission links; this loss of power varies both with the change of power on the link and with the power being carried at the time of the change. The power system may be complex, and the network configuration may change from day to day. To estimate the cost of the change in system transmission loss for any given change in power conditions is therefore difficult, particularly under operating conditions.

It is well known that a network analogue of a power-transmission system may be constructed in which the impedance of each 3-phase link is represented by a single resistor and power is represented by current. If the impedance of a transmission link is represented by the total resistance of two or more resistors in series, one of these resistors can represent the resistance of the link. The voltage appearing across the resistor may be multiplied by any chosen factor (if the analogue is fed with alternating current) by means of a transformer, and the outputs of such transformers may be added together. A measure of the change in system transmission loss has been obtained by the use of such a network analogue. The analogue has been incorporated into a computer which evaluates the cost of any change of power in a given power system, and so enables the most economical allocation of generation to be determined.

(2) THEORETICAL BASIS OF THE COMPUTER

(2.1) Evaluation of the Change of Transmission Loss due to a Change in Power Transmitted

It is assumed that the current flowing in a transmission link is proportional to the power transmitted. The power lost in a link is then proportional to the square of the power transmitted.

If the power in a transmission link n is P_n, the power lost in the link is $K_nP^2R_n$, where K_n is a constant and R_n is the resistance of the link. Subsequent to a change in power ΔP_n, in the same direction as P_n, the power loss is $K_n(P_n + \Delta P_n)^2R_n$. The change in transmission loss due to the power change ΔP_n is therefore $K_nR_n[2P_n\Delta P_n + (\Delta P_n)^2]$. If the change is small, say within 10% of the load P_n, the term in $(\Delta P_n)^2$ may be neglected.

In the computer which has been constructed, the term in $(\Delta P_n)^2$ for the transmission loss has been ignored. The constant K_n has been assumed to be the same for all links, corresponding to a constant voltage over the entire system and a constant power factor in all links. Nominal voltage and unity power factor

Written contributions on papers published without being read at meetings are invited for consideration with a view to publication.
Mr. Wardrop is in the Electrical Engineering Department, Imperial College of Science and Technology, University of London.

have been assumed. It seems probable that the transmission loss so determined will be lower than that occurring in the power system. A computer could be constructed so that K_n could be chosen independently for each link, and so increase the accuracy of the determination.

(2.2) Evaluation of Costs

The computer evaluates the cost of power changes so that the change involving the least cost may be found. If changes in generation are considered, it is necessary to take account of the marginal generation costs, and these are assumed to be constant for the changes of generation considered. It is also assumed that the change in transmission loss will be supplied by the generating station where an increase in output is made.

The incremental and decremental generation costs at station α (the cost and saving, respectively, in pence per megawatt-hour generated or reduced) are Δd_α and ∇d_α. If the change of transmission loss (in megawatts) due to generating 1MW at station α rather than at station β is $\Delta\lambda_{\alpha\beta}$, the cost of generating 1MW at α rather than at β is given by

$$\Delta d_\alpha - \nabla d_\beta + \Delta\lambda_{\alpha\beta}\Delta d_\alpha \text{ pence/hour}$$

The cost of supplying an increase in load of 1 MW on the system from station α is given by

$$\Delta d_\alpha(1 + \Delta\lambda_\alpha) \text{ pence/hour}$$

where $\Delta\lambda_\alpha$ is the change of transmission loss in megawatts per additional megawatt generated at station α. To evaluate $\Delta\lambda_\alpha$ the location of the increase in load must be known or presumed.

(3) CONSTRUCTIONAL DETAILS

An electrical model of the power system for which the cheapest allocation of generation is to be found forms the basis of the computer. The power system shown in Fig. 1 was chosen

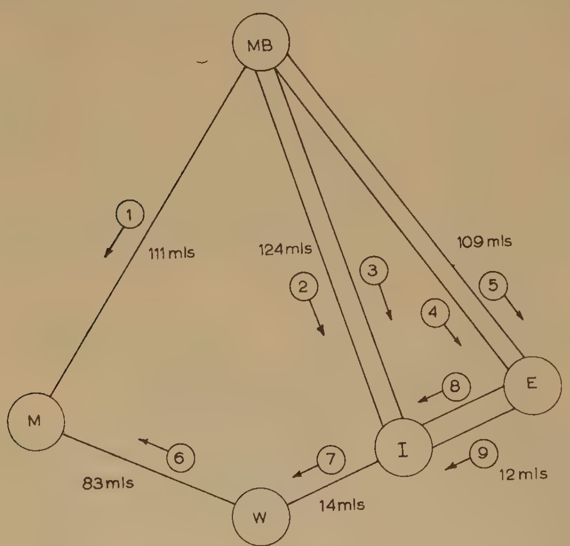

Fig. 1.—Power system represented.

O→ Indicates circuit reference number and positive direction of power flow
Impedance of each circuit = $0{\cdot}055 + j0{\cdot}55$ ohm/mile.

because power and cost data were available for it. Each branch of the model network is composed of three resistors in series: the first provides for the measurement of current in the branch, the second represents the resistance of the transmission link, and the third makes the total resistance of the branch proportional to the impedance of the transmission link, as shown in Fig. 2.

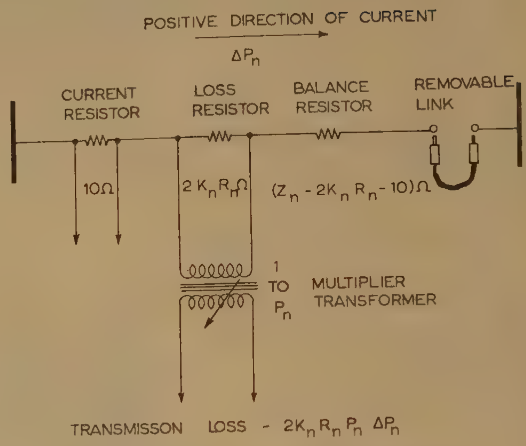

Fig. 2.—Branch n of network analogue.

Current is injected and withdrawn from the model network t[o] represent any change of power on the system. A current corre[-] sponding to ΔP_n then flows in branch n of the model network giving a voltage of $2K_nR_n\Delta P_n$ across the second resisto[r]. This voltage is fed to a variable-ratio multiplier transformer, th[e] ratio being set to correspond to the load, P_n, on the trans[-] mission link prior to the change. The output voltage of thi[s] transformer, $2K_nR_nP_n\Delta P_n$ represents the change of power los[s] in the link. The output voltages of all such multiplier trans[-] formers are added to give the change in system transmission los[s]. Incremental and decremental generation costs are represente[d] on the computer by transformer ratios which multiply a voltag[e] proportional to the change in generation to give a voltage pro[-] portional to its cost, ignoring transmission losses. The voltag[e] representing the change of transmission loss is also multiplie[d] by the use of a transformer having a ratio representing generatio[n] cost to give a voltage proportional to the cost of the loss. Thes[e] two 'cost' voltages are added to give a measure of the cost o[f] the proposed change in power conditions, as shown in Fig. 3.

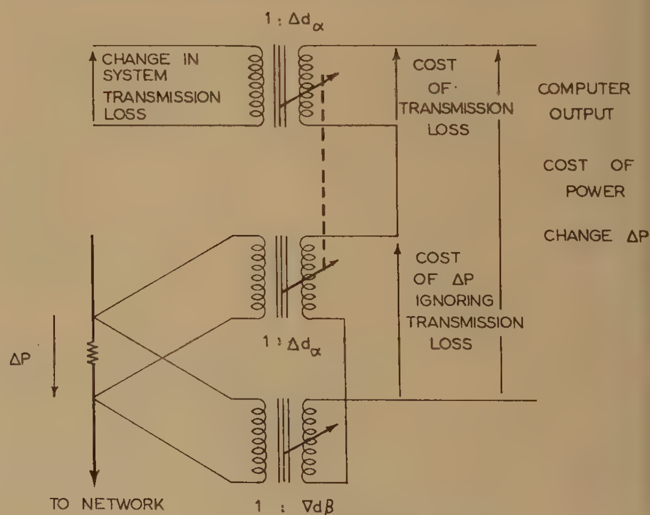

Fig. 3.—Computation of cost.

Sequential calculation of the cost of changes of power is carri[ed] out by providing a cost transformer for each power station, a[nd] making the requisite connections via a continuously-driven u[ni]selector. The minimum computer output voltage in the sequen[ce] is detected, and the change involving the minimum cost indicate[d].

Voltages are measured by a valve voltmeter, and their sign

indicated either by the slope of a Lissajous figure on an oscillograph or by a phase-sensitive detector. Voltages in phase with the supply voltage are considered positive, and those in anti-phase are considered negative.

(4) COMPUTER OPERATION

Information which may be used to minimize the cost of supplying power may be obtained by operating the computer in any of the following ways:

(*a*) To obtain the change in the total cost of supplying power resulting from a change in location of generation or load.
(*b*) To obtain the change in the total cost of supplying power resulting from a change of power demand on the system.
(*c*) To determine the cost of generation at any location in a power system knowing the cost at one given location.
(*d*) To determine the power flows in all transmission links immediately subsequent to any link being switched into or out of service.
(*e*) To determine the power flows and transmission losses in the transmission links for any given specification of generation and load at the supply points of the system.
(*f*) To evaluate the term in $(\Delta P_n)^2$ which is neglected in the determination of transmission loss by means of the computer.

The model network is arranged to have the same configuration as the actual power system for all methods of operation. Although all power flows, losses and costs are available on the computer, very few quantities are significant during any particular evaluation and so many alternatives may be compared rapidly.

(4.1) Cost of Change in Location of Generation or Load

The computer is operated to find the cost of transferring x megawatts of generation to station α from station β, with station α supplying the consequent change in transmission loss. The total power supplied by the system is assumed to be constant. A change in location of load is equivalent to a change in location of generation, and its cost is evaluated in a similar manner.

It is assumed that the power carried by each link in the power system is known prior to the proposed change. This would be given by system metering for a present operating condition, or by calculation for future conditions. In either case the ratios of the multiplier transformers are set to correspond to these line flows in both magnitude and direction. Current, corresponding to x megawatts, is supplied to the network at α and withdrawn at β, as shown in Fig. 4(*a*). The ratios of the cost transformers are set to correspond to the incremental generation cost at α and the decremental generation cost at β. The computer then gives a voltage output proportional to the cost of the change. A negative voltage indicates that the cost of supplying power will be reduced by transferring generation to station α from station β.

(4.2) Cost of General Increase in Power Demand

The computer is operated to find the cost of supplying an increment of x megawatts of load from alternative power stations. The station supplying the increment of load also supplies the change in transmission loss. The multiplier transformer ratios are set to correspond to the power on each link of the power system prior to the increment of load. Current, corresponding to x megawatts, is supplied to the network at one of the power stations and withdrawn via resistors at every busbar where an increment of load is assumed, as shown in Fig. 4(*b*). The incremental generation cost at the proposed station is set on the Δd_α controls of the cost transformers and the ∇d_α controls are set to zero. The output voltage of the computer is then proportional to the cost of supplying the increment of load from the station. The computation is then repeated for each alternative station to discover the station which can supply the increment of load at minimum cost.

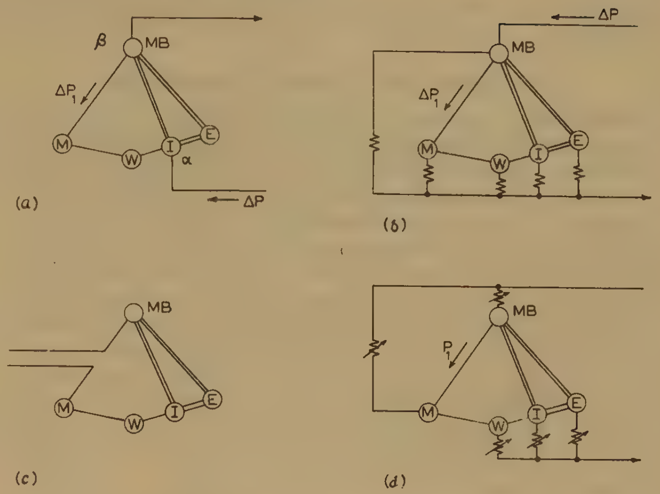

Fig. 4.—Methods of operation.

(*a*) Change of location of power.
(*b*) General increase in power demand.
(*c*) Line switching.
(*d*) Power distribution.

(4.3) Transfer of Generation Cost from one Location to Another

The cost of generation is usually evaluated for power supplied to the high-voltage busbars of a generating station. The cost required for load-dispatching purposes may be the cost at another busbar, and the computer may be used to evaluate this cost. The multiplier transformer ratios are set to correspond to the powers flowing in the lines during the period when power is to be transferred between the two locations. A current corresponding to 100 MW is injected into the network where the incremental generation cost is known at busbar α, and withdrawn where the cost is required to be known at busbar β, as shown in Fig. 4(*a*). The Δd_α controls of the cost transformers are set to the known incremental generation costs in pence per megawatt-hour and the ∇d_α controls are set to zero. The computer then gives a voltage proportional to the incremental generation cost when supplying power at busbar β from station α.

(4.4) Effect of Line Switching

The computer can be used to determine the effective or 'Thévenin' impedance of the system between circuit-breaker contacts before they close. The voltage across the open circuit-breaker contacts may be divided by this impedance to give a measure of the current which will pass through the contacts when the circuit-breaker is closed. To determine this system impedance, current is fed into the model network as shown in Fig. 4(*c*). The computer may be used to determine the redistribution of power in a power system network subsequent to a line being switched out of the system. System loads and generator outputs may be taken as constant for a short period following a line outage. The distribution of power in the network during this period may therefore be determined by superimposing the power distribution obtained for a power flow in the line to be switched out of service equal and in the opposite direction to the original flow, with all generators and loads disconnected from the system. The computer is used to determine these changes in power flow in all lines due to the switching out of one line. Current is injected into that line on the network in the opposite direction to the original power flow, as shown in Fig. 4(*c*). The currents flowing in the other lines represent the changes in power

that will occur when the line under consideration is switched out of service.

(4.5) Determination of Power Flows and Losses

Powers on a power system are usually specified by the magnitude of generation and load connected to each supply point or node of the system network. Only limits of power and voltage are specified for the links in the system. Although the computer has been designed primarily to evaluate the effect of changes in power, it can be used to determine the distribution of power in the system for any specified loading condition. Generation is represented by current injected into the network and load by current taken from the network. These currents are controlled by variable resistors connected between the supply and the network, as shown in Fig. 4(*d*). Power flows in the links are represented by currents flowing in the model network, and if the multiplier transformer ratios are set to correspond to these power flows, the computer output voltage gives a measure of the system transmission loss.

(4.6) Calculation of the $(\Delta P_n)^2$ Term

In evaluating the change in transmission loss the term $K_nR_n(\Delta P_n)^2$ was neglected to simplify the construction of the computer. If for any change in loading conditions the multiplier transformers are set to correspond to the values of the change of power, ΔP_n, the computer voltage represents twice the transmission loss due to the $(\Delta P_n)^2$ term. The error loss is always positive, and is constant for any given change of power, independent of the original powers flowing in the network.

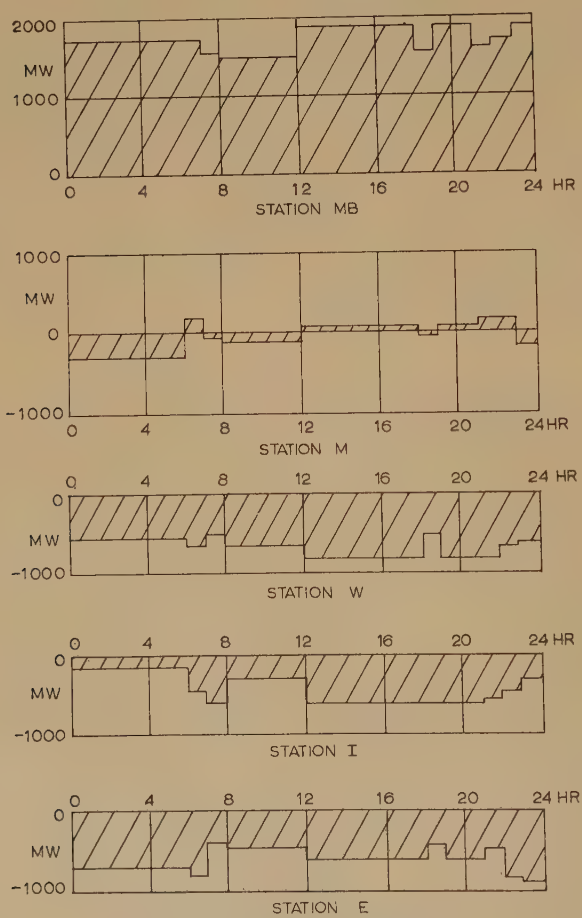

Fig. 5.—Specified generation.

(5) TEST RESULTS

To illustrate the use of the computer, a series of tests were carried out using estimated future power and cost data. Complete details of all line power flows and losses were recorded, as well as system power loss and cost changes, for all tests. To check the accuracy of the construction and operation of the computer, several results were evaluated by using a desk calculating machine. The error in any measured value was within $\pm 3\%$ of that value.

(5.1) Specified Data

The specified power delivered to each station of the power system for each hour of the day is shown in Fig. 5. The computer was used to determine the flows of power in each link of the system, and these are shown in Fig. 6, the sign of power

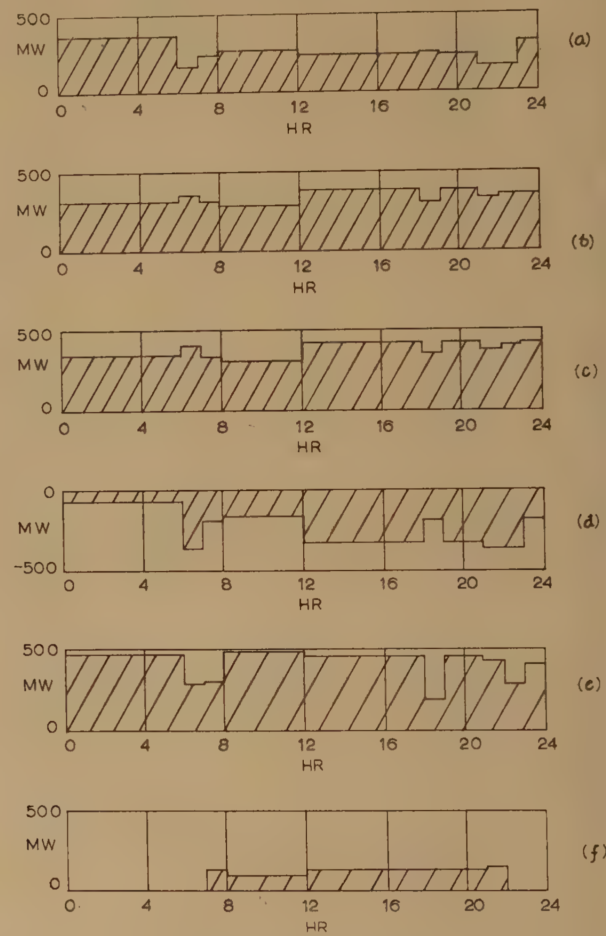

Fig. 6.—Line power flows.

(*a*) Line 1.—MB to M.
(*b*) Lines 2 or 3.—MB to I.
(*c*) Lines 4 or 5.—MB to E.
(*d*) Line 6.—W to M.
(*e*) Line 7.—I to W.
(*f*) Lines 8 or 9.—E to I.

flow corresponding to the positive direction of flow as show by arrows on the network in Fig. 1. The incremental a decremental costs of generation at each station were also specifi for each hour of the day. It was assumed that no allowance f transmission losses had been made in the specified data.

(5.2) Change in Location of Generation

The effect of transferring 100 MW of generation to statio from station MB was studied. At no time would any circuit

overloaded by the transfer, and the transmission loss and cost due to it are given in Fig. 7. The transfer would result in a reduction in system transmission loss of about 6 MW throughout the day. The cost change is variable between an increase of

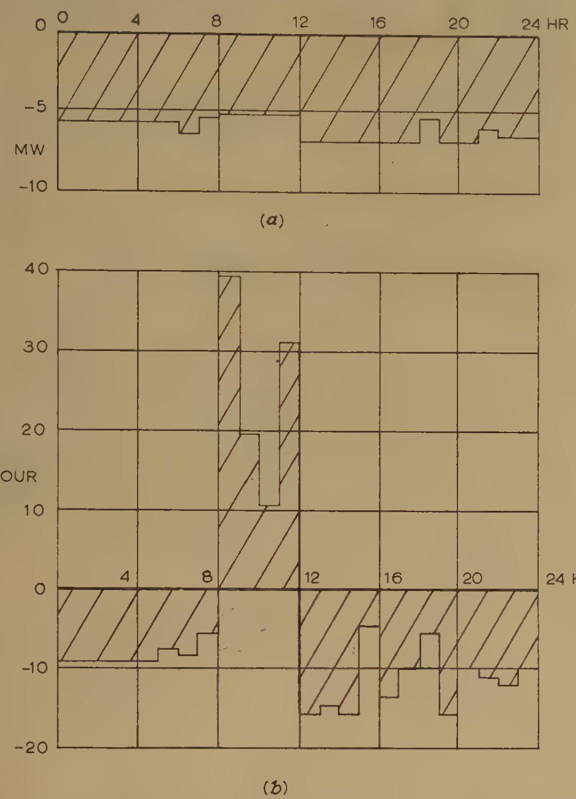

Fig. 7.—Transmission loss and cost due to a transfer of 100 MW to Station I from station MB.

(*a*) Transmission loss.
(*b*) Cost.

£40 and a reduction of £15 per hour. If 100 MW were transferred to station I from station MB during the periods of cost reduction, i.e. between 0000–0800 and 1200–2400 hours, a daily reduction of £200 in the cost of supplying power would be effected.

(5.3) General Increase in Load

The costs of supplying a general increase in load of 100 MW from stations MB, M or I were determined by use of the computer. The increment of load was distributed in the following manner: MB, 46 MW; M, 7·8 MW; W, 15·4 MW; I, 15·4 MW; and E, 15·4 MW.

Since station I had the lowest incremental generation cost for the majority of the day and the system transmission loss was reduced by any increment of generation at station I, generation at that station was the cheapest irrespective of the transmission loss.

(5.4) Evaluation of the $(\Delta P_n)^2$ term

The computer was used to evaluate the $(\Delta P_n)^2$ term in the expression for transmission loss which is neglected in the normal operation of the computer. The loss due to this term is 0·18 MW for the test described in Section 5.2 and corresponds to about 3% of the change in system loss for that test.

(6) CONCLUSIONS

The example chosen to illustrate the use of the computer showed a daily saving of £200 to be possible by taking proper account of the transmission loss in load dispatching. The total system transmission loss was 3% of the total power transmitted, and cost £3 000 per day, based upon 0·5d/kWh.

The amount of money to be saved by using the computer is highly dependent upon the difference between the marginal generation costs at alternative power stations. If this difference is greater than 10%, it is likely that the station with the lowest marginal cost will be the cheapest station to operate, irrespective of the transmission loss.

The cost of the computer is low, for it is composed of relatively cheap components—resistors, wafer switches and multi-ratio transformers. If similar equipment exists, to calculate the effect of line switching, for example, its extension to a power-cost computer is a simple matter. The computer described in the paper may be operated manually or incorporated into a scheme of mechanized load-dispatching.

(7) ACKNOWLEDGMENTS

The author wishes to thank the Central Electricity Generating Board for permission to publish the paper, and for financial support. He is indebted to his colleagues for much help and encouragement.

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The Institution of Electrical Engineer
Paper No. 290
Apr. 195

A REVIEW OF WORK TOWARDS NUCLEAR ENERGY FROM CONTROLLED THERMONUCLEAR REACTIONS

By D. W. FRY, M.Sc., Member.

(*Lecture delivered at an Ordinary Meeting of* The Institution *held in conjunction with the* British Nuclear Energy Conference, *4th December*, 1958.)

(1) INTRODUCTION

At the Second United Nations Conference on the Peaceful Uses of Atomic Energy, held in Geneva during September, 1958, one of the main topics discussed was the work in progress in several countries aimed at releasing nuclear energy from the elements found at the beginning of the periodic table. About 115 papers were presented in this field, and they will soon be widely available; within the scope of this lecture it will thus be impossible to do more than give the barest outline of the main approaches being made to the problem and summarize the stage which has been reached.

The early work on reactions between nuclei of the light elements goes back to the historic experiment carried out by Cockcroft and Walton[1] at the Cavendish Laboratory in 1932. They used protons to bombard a lithium target and demonstrated that a proton-lithium nuclear reaction took place. Many studies of reactions between the nuclei of the light elements were made from 1932 onwards, and some of the reactions which are of most interest for the thermonuclear work are listed in Table 1. There

Table 1

Fusion Reactions

$D + D \rightarrow He^3 + n +$	3·25 MeV
$D + D \rightarrow T + p +$	4 MeV
$T + D \rightarrow He^4 + n +$	17·6 MeV
$He^3 + D \rightarrow He^4 + p +$	18·3 MeV
$Li^6 + D \rightarrow 2He^4 +$	22·4 MeV
$Li^7 + p \rightarrow 2He^4 +$	17·3 MeV

are two possible deuterium-deuterium reactions, one giving tritium and a proton, the other forming He^3 and a neutron; the probabilities with which these two reactions occur are approximately the same. A third reaction of interest is that between deuterium and tritium, the reaction products being He^4 and a neutron with the release of 17·6 MeV of energy.

(2) FUNDAMENTAL IDEAS AND PROBLEMS

If every deuterium nucleus present in an accelerated beam of deuterium ions were to make a nuclear collision when the beam struck a liquid deuterium target, there would be no difficulty about achieving an energy gain. Unfortunately a very large proportion of the deuterons in the beam will lose their energy in the target by interaction with the screening electrons without a nuclear collision taking place. In the Cockcroft–Walton experiment only one proton in 10^8 striking the target made a nuclear collision. In principle, this loss could be avoided by firing two beams of energetic deuterons at each other, but it is a simple matter to show that the ion-beam densities required to make this a useful method would be impractically high. The alternative approach is to use a hot gas. At the temperatures necessary for the reaction rate to be useful, deuterium gas is fully ionized. The electrons and ions are completely dissociated, but both will be present in the reaction vessel and charge neutralization will normally take place to a high degree. A hot gas in this state is termed a plasma. The energetic ion will, in general, make many collisions, depending on the ga temperature, before undergoing a nuclear reaction.

Energy can be lost from the hot gas by radiation and by th escape of energetic particles to the walls. The radiation i primarily by the process known as *Bremsstrahlung*, i.e. th radiation from energetic electrons when they change directio as they pass through the electric fields of the nuclei. In any ho plasma which can be established on a laboratory scale the radi ation will leave the system without scattering, and the Stephan Boltzmann T^4 law will not be applicable. The intensity c radiation can be established only by detailed consideration of th process. Cillie,[2] Thompson[3] and Spitzer[4] have shown that th power radiated per unit volume is

$$P_R = 1{\cdot}4 \times 10^{-27} Z^2 n_i n_e T^{1/2}\ \text{ergs/cm}^3\text{/sec} \quad . \quad . \quad ($$

where Z is the atomic number, n_i and n_e are the numbers of ior and electrons per unit volume, and T is the absolute temperatur

It is of interest to note that the radiation is proportional t Z^2 per electron-ion collision. This shows the need to keep th number of impurity atoms present in a deuterium gas ve small, in order to avoid excessive radiation losses.

Calculations have been made of the power released in h plasmas of deuterium and a deuterium-tritium mixture a function of temperature, and curves for a density $n = 10^{17}$ nuclei/cm^3 are shown in Fig. 1; also shown is a cur

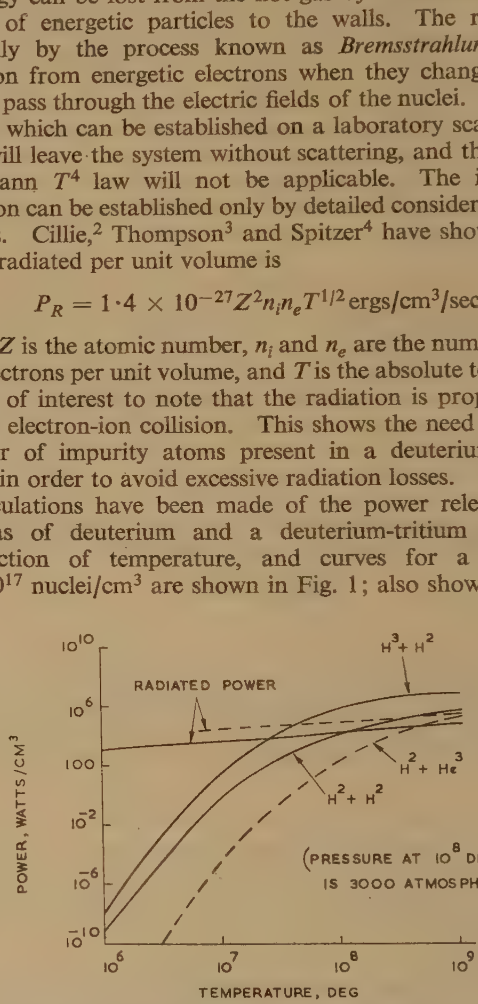

Fig. 1.—Variation of power release with temperature.

for the power radiated as *Bremsstrahlung* from deuterium g Lawson[5] considered the power balance of a pulsed system which the gas was heated for a time t; to be of use the ener released during this time must be greater than the sum of t energy given up as radiation and the energy fed into the gas wh is returned to the walls, and Lawson showed that the ratio betwe the power released and that supplied is proportional to $n_i t$.

When $n_i t \simeq 10^{16}$ the temperature required for a useful reac using a deuterium-deuterium reaction is about 3×10^8 deg while for a tritium-deuterium system, where conditions

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easier but still severe, it is about 5×10^7 deg K. Thus the main problems facing the physicist and the engineer are to heat the ions to the required temperature, and to isolate the hot gas from the walls of the reaction vessel for a period long enough to ensure that a sufficient fraction of the nuclei present make nuclear collisions and so release more energy than is supplied.

All the devices being studied rely on a magnetic field to hold the hot gas away from the walls of the containing vessel. In principle, a magnetic field reduces the flow of charged particles across it, but the flow of particles along the magnetic field lines remains unhindered. In order to confine a hot plasma, either the magnetic lines must close within the containing vessel or a means must be provided to prevent leakage along them.

(3) CLOSED-LINE CONTAINMENT SYSTEMS

(3.1) The Pinch

(3.1.1) Principles.

The pinched discharge is a closed-line system which has received attention in several laboratories, and the self-constricting properties of a current passing through an electrolyte were first described by Northrup in 1907.[6] In a pinched discharge the ionized gas contracts through the action of the magnetic field which accompanies the current flowing through it. In a straight tube there is radial containment only, but the end losses may be eliminated by making the discharge tube a toroid.

The magnetic field, H_θ, exerts an inward pressure given by $H_\theta^2/8\pi$, and the outward gas pressure is nkT, where n is the particle density, k is Boltzmann's constant, and T is the absolute temperature. The ratio between these two pressures is denoted by β:

$$\beta = \frac{nkT}{H_\theta^2/8\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In a simple pinched discharge the particle pressure balances the pressure due to the self magnetic field. Thus, for a uniform plasma of radius r, $H_\theta = 2I/10r$, and $N = \pi r^2 n$; inserting these values in eqn. (2) with $\beta = 1$ gives

$$I^2 = 2 \times 10^2 NkT \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where I is in amperes and N is the number of particles per unit length of the column.

This important relationship was first obtained by Bennett.[7] For a given value of I and N the temperature is thus independent of the radius of the discharge. Experiments indicate that the value of n with which it is necessary to operate lies in the region of 10^{14}–10^{18} particles/cm^3. With a pinch radius of 10 cm, currents of 10^6–10^7 amp would be necessary to attain the required temperatures. In studies of pulsed processes of short duration currents up to 10^6 amp and rates of rise of 10^{10}–10^{11} amp/sec with densities in the region of 10^{15} particles/cm^3 have been used. When its rate of rise is rapid, current starts flowing through the gas in a thin cylindrical shell close to the tube wall, owing to the skin effect. As the current builds up it contracts inwards, owing to the presence of the self magnetic field, carrying the ionized gas with it. The radial velocity of propagation is usually supersonic, so that the ionized gas becomes compressed into a thin wall.[8–11]

Studies of pinched discharges soon showed that a highly constricted column of hot gas is unstable against forces which distort its shape. It is easy to illustrate that this is so on physical grounds. A current flowing along a straight wire is accompanied by magnetic lines of force which form rings in planes perpendicular to the wire, with their centres on its axis. A small local disturbance of the wire will result in an increase in magnetic field on the concave side of the disturbance, the result of which is to increase the initial disturbance. Theoretically the column may have an infinite number of modes of instability, and two[12] of these are shown in Fig. 2. In the ordinary dynamic pinch it is theoretically possible[13, 14] to obtain a positive energy gain before the instabilities destroy the pinch, but, apart from considerable practical difficulties, the resultant energy release would be on an explosive scale.

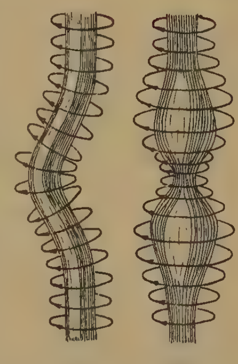


Fig. 2.—Instability modes in a pinched discharge.

It may be seen intuitively that a moderate amount of longitudinal magnetic field should provide some degree of stabilization. If an axial magnetic field fills the tube initially, it will be trapped and compressed by the highly ionized shell of hot gas travelling inwards. These trapped magnetic lines form a back-bone for the discharge, and the stabilizing effect of a longitudinal magnetic field applied either inside or outside the pinch column has been analysed.[15–18] Theory suggests that the very fast-growing sausage-type instability is suppressed by a moderate amount of longitudinal magnetic field. The next mode, or corkscrew instability, then tends to predominate, and theory predicts that this can be controlled within limits if the discharge is contained in an external conducting shell. The main condition predicted theoretically as necessary for complete stability is that the longitudinal magnetic field remaining outside the pinch should be small compared with that which is trapped.

The emphasis on separation of the H_Z and H_θ fields underlines the importance of producing a very highly conducting shell between the two fields, for the H_θ field is determined by the current flowing in the plasma, and this must disappear as the pinched plasma escapes from containment. One of the main objectives of the pinch work is to create very quickly a highly conducting current shell which will contract and in doing so trap and retain within it, for longer times than have yet been achieved, the H_Z field. When the axial H_Z field is trapped there is an outwards magnetic pressure in addition to the gas pressure, with the result that β must be less than unity.

If the rate of current build-up is about 10^8 instead of the 10^{10} amp/sec or more which has been used in much of the fast pinch work, then different dynamic conditions may be expected. In particular, with the slower rate of compression there will be negligible heating by shock-wave processes. Instead, the current will be distributed throughout the column of the gas and Joule heating will take place in the same way that a wire carrying a current is heated. However, instabilities are still to be expected. The stability theory for pinches with volume current distributions[15–17] is in a less advanced state than is the case where the current might be considered as being on a surface. Up to the present, no improvement in stability has been predicted for the slower pinches, and the theories are unable to explain the plasma confinement which has been observed in Zeta and Sceptre.

(3.1.2) Experimental Installations and Results.

Seven linear pinch installations and eight using toroidal tubes were described at Geneva, and their characteristics are summarized in Table 2. The peak discharge currents used vary from 0·05 to 1·0 MA, the plasma densities from 10^{14} to 7×10^{17} ions/cm^3, and the pulse time from a microsecond to a millisecond; the electron 'temperatures' (usually estimated) have been in the region of 10^5–10^6 deg K and the ion 'temperatures' between 1 and 5×10^6 deg K. In most cases 'temperature' needs to be in quotation marks, because it is estimated

Table 2

Fusion Devices Described at Geneva

Classification	Device	Location	Dimensions: Bore	Dimensions: Diameter or length	Discharge current	Magnetic field	Plasma density	Discharge time	Temperatures: Electron	Temperatures: Ion
			cm	cm	mA	kG	ion/cm³ $\times 10^{16}$	sec $\times 10^{-5}$	deg K $\times 10^5$	deg K $\times 10^5$
Toroidal stabilized pinch	Uppsala torus	Sweden	28	130	~1·0	—	—	—	—	—
	Saclay torus	France	8	78	~0·05	—	1·0	10	—	—
	Alpha	U.S.S.R.	150	450	~0·2	—	Believed similar to Zeta			
	Zeta	U.K.	100	300	0·2	—	0·01	100	1–5	50*
	Perhapsatron S4	U.S.A.	14	70	0·3	—	0·5	1	1	10†
	Gamma pinch	U.S.A.	10	60	0·3	—	1·0	—	5	—
	Sceptre	U.K.	30	110	0·2	—	0·1	10	1	30‡
	Moscow torus	U.S.S.R.	48	125	0·2	—	0·04	2	1·5–3	—
Linear pinch	Moscow linear pinch	U.S.S.R.	40	50	~1·0	—	70	0·1	5–10	10‡
	Uppsala linear pinch	Sweden	30	60	0·4	—	—	—	—	—
	Munich linear pinch	Germany	20	50	0·4	—	10	0·1	—	~10
	Columbus II	U.S.A.	10	30	0·8	—	—	0·1	—	—
	Columbus S4	U.S.A.	13	60	0·25	—	5	0·5	1	—
	Columbus T1	U.S.A.	15	600	0·07	—	0·1	100	0·3	0·1
	Maggi	U.K.	15	28	0·5	—	30	0·1	—	—
	Saclay linear pinch	France	28	100	0·2	—	—	—	—	—
	Triaxial pinch	U.S.A.	10	100	1·3	—	5	0·5	—	—
	Screw dynamic	U.S.A.	10	20	0·25	— 30	5	0·1	—	—
Stellarator	B1	U.S.A.	5	450	—	30	0·003	100	—	—
	B2	U.S.A.	5	600	—	—	—	—	—	—
	B3	U.S.A.	5	600	—	50	—	—	5	—
	B65	U.S.A.	15	500	—	20	0·001	100	—	—
	C	U.S.A.	20	1 200	—	50	Not yet completed			
Astron	Pilot model	U.S.A.	100	1 000	—	1·6	0·0001	—	1–3 MeV electron injection	
	Final power reactor	U.S.A.	200	2 000	—	40	0·007	—	50 MeV electron injection	
Mirror or adiabatic trap device	D.C.X.	U.S.A.	50	50	—	10	600 keV molecular ion injection			
	Ogra	U.S.S.R.	140	2 000	—	5	200 keV molecular ion injection			
	Felix	U.S.A.	45	270	—	25	10–15 keV molecular ion injection			
	High compression	U.S.A.	15	100	—	44	0·01	1 000	1 000	100
	Scylla	U.S.A.	7	7	—	38	~10	~0·1	—	—
Rotating plasma	Ixion	U.S.A.	24	86	—	10	0·01	100	—	—
	Homopolar	U.S.A.	25	10	—	13	1·0	1	—	—

* By Doppler broadening of impurity light. † By magnetic pressure balance. ‡ From piezo-electric measurements.

either from a knowledge of pressure, density, magnetic field, etc., or from a spectroscopic measurement which is unable to distinguish between thermal motion and rapid irregular motion of the plasma.

It may be shown that in the fast linear pinch the temperature at the first maximum contraction is proportional to dI/dt, and for an LC circuit the initial rate of rise of current is given by

$$\frac{dI}{dt_0} = \frac{V_0}{L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where V_0 is the starting voltage and L the total circuit inductance. To obtain a high temperature by rapid compression[18] demands that V be high and L low. Two installations have been built at the Atomic Weapons Research Establishment, and Table 3 gives some of their characteristics. The main features of the circuit used have been the reduction of the inductance by capacitor design and by multiple parallel switching. The main problem here is to fire the large number of spark-gaps used as switches in a time which is short compared to the transit time of pulses through the cables connecting them to the common load. The trigger spark-gap used was of very low inductance, having a spacing of approximately 2 mm and a working pressure of

Table 3

Characteristics of A.W.R.E. Fusion Devices

	6 kJ bank	*45 kJ bank*
Nominal capacitance, μF	121	100
Number of units	109	200
Nominal working voltage, kV	10	30
Inductance (without tube), mμH	8·50	5·5
Inductance of condenser and spark gaps, mμH	1·31	1·8
Inductance of cables, mμH	3·64	2·0
Inductance of collector plates, mμH	3·55	1·7
Short-circuit dI/dt, amp/sec	$1·2 \times 10^{12}$	$5·5 \times 10^{12}$
Short-circuit current, MA	1·1	3·6
Peak trigger voltage, kV	20	30
Trigger-voltage rise time, millimicrosec	15	15

several hundred pounds per square inch. The main-bank spar gaps worked at atmospheric pressure, and the statistical lag w kept within 10^{-9} sec by careful attention to design. The di charge tube used on the small bank was 28 cm long and 15 c bore.

Installations using a slow rate of current rise are Zeta, Sceptre[20] and Alpha.[21] In Zeta (Fig. 3) the current builds u to its maximum value in 1 millisec and the duration of the pul

Fig. 3.—General view of Zeta.

is about 5 millisec; the basic circuit is a capacitor discharging through a mechanical switch into the primary circuit of an iron-cored transformer, the secondary being the gas in the torus. The four main components are:

(*a*) A 1 600 μF 25 kV capacitor bank designed for unidirectional voltage operation under maximum load conditions.
(*b*) An iron-cored transformer capable of handling 7 volt-sec/turn at each pulse.
(*c*) An aluminium torus with a bore of 1 m and a mean circumference of 12 m fitted with a liner system to reduce power arcing.
(*d*) Windings on the torus to provide a magnetic field up to 1 kG parallel to the gas current.

Currents in excess of 0·2 MA flowing for 3–5 millisec were measured. Both inductance and magnetic search-coil measurements showed that the current channel was constricted into the centre of the torus. It was observed that the magnetic-field fluctuations increased as the contaminant gases present in the system decreased, suggesting that the plasma became more unstable. The observed distribution of magnetic fields has yet to be explained.

(3.1.3) **Present Status of the Pinch Programme.**

All experimenters have now observed neutrons to a varying degree. While the conditions for production vary somewhat, no one has yet claimed that they have arisen from nuclear reactions in a plasma in which the particles are in thermal equilibrium. Observations on both fast and slow pinch systems have shown that a large unaccountable energy loss is always present. The cause of this is being investigated.

Some evidence[12, 22] has been produced that large numbers of electrons may be gaining a large amount of energy and escaping across the magnetic fields. However, much more needs to be known about the loss mechanism before we can be certain of it and able to say whether it can be avoided by a suitable choice of operating conditions.

(3.2) The Stellarator

(3.2.1) **Principles.**

Stellarator is the name which has been given to the system being developed at Princeton University by Prof. Spitzer and his colleagues. It differs from the pinch approach in that it relies on a strong externally applied magnetic field to provide containment of the gas; no H_θ field due to a plasma current need be introduced for this purpose.

A study of the motion of the electrons and ions in the plasma shows that in a uniform field they move in helical paths around the lines of the magnetic field. Thus we have the concept of a plasma being tied to the lines of the magnetic field. In a torus system the magnetic-field lines are circles centred at the axis of symmetry, so that the field distribution over the cross-section of the torus bore is non-uniform (Fig. 4). Gunn,[23] Alfven[24] and others have shown that when the field is non-uniform in a radial direction there is an axial drift of the particles across the magnetic field lines. This drift is in opposite directions for positive and negative particles, giving rise to charge separation and a resultant electric field across the plasma. The combination of this electric field and the external magnetic field results in the plasma drifting outwards, and equilibrium stability cannot be achieved.

(3.2.2) **The Rotational Transform Concept.**

Spitzer and his colleagues considered the motion of particles when a magnetic line was displaced through an angle θ instead of being closed on itself after one revolution (Fig. 5). Then the

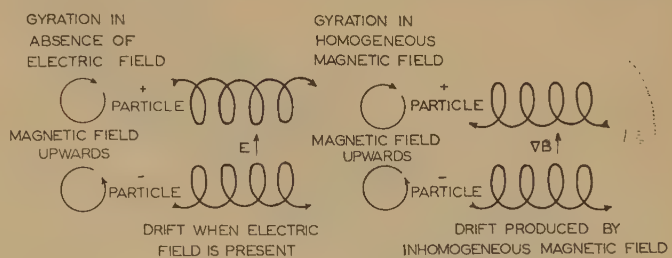


Fig. 4.—Field distribution in torus.

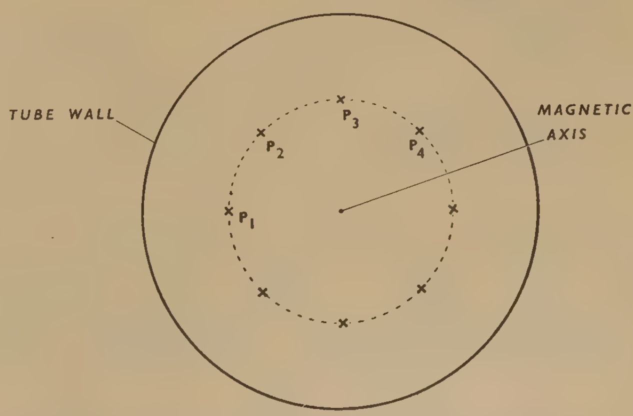


Fig. 5.—Rotational transform cross-sectional plane, showing intersection points produced by a single line of force.

magnetic lines will almost always generate a magnetic surface. The result of this transformation is that the position of a line changes and connects regions of positive and negative charges, so that they can neutralize each other by movement along the field lines.

Several ways have been proposed for achieving this transformation in practice, one being to twist a torus into a figure 8, as shown in Fig. 6. The same effect may be produced by the use

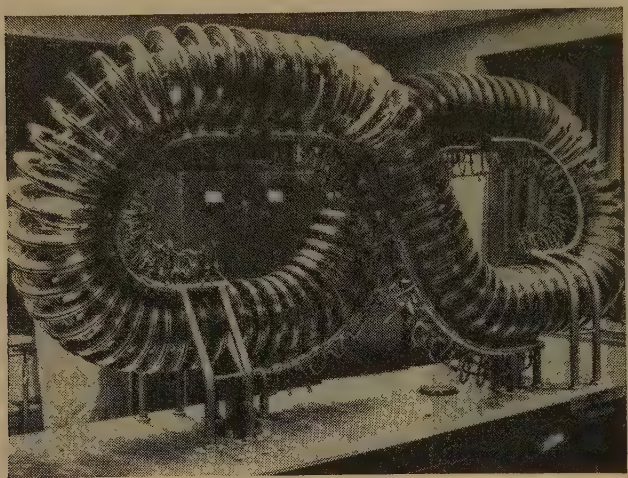
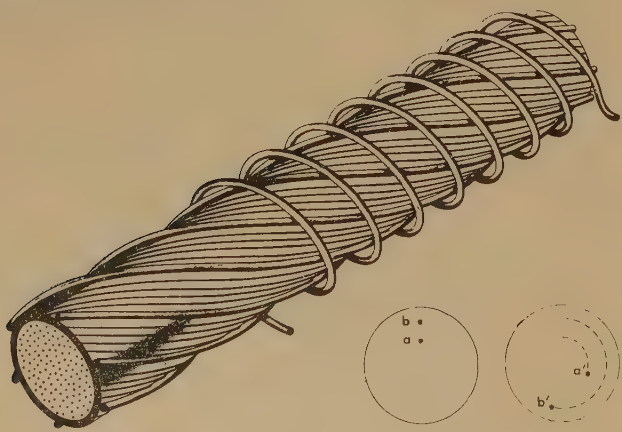


Fig. 6.—Torus of the figure-8 Stellarator.

of a transverse magnetic field whose direction rotates with distance along the magnetic axis. This field pattern may be produced by coils wound helically on the walls of a simple torus, the currents in adjacent turns of the helices flowing in opposite directions, as shown in Fig. 7.



(*a*) (*b*) (*c*)

Fig. 7.—Stabilizing windings on the Stellarator.

(*a*) Lay of windings.
(*b*) Two illustrative lines of force through points *a* and *b*.
(*c*) Relative positions of these two lines further along tube.

If a circulating current is used to heat the gas, the azimuthal magnetic field due to the current also introduces a transform angle. Kruskal[25] pointed out that the plasma would be unstable when the circulating current exceeded the value for which the transform angle was reduced to zero (or increased to 2π), and there is experimental evidence[26] for 'kink' instability at currents above the 'Kruskal limit'.

(3.2.3) Heating.

A current of this limiting value or less may be used to provide Joule heating of the plasma. However, its conductivity is proportional to $T^{3/2}$ and above about 10^6 deg K the rate of energy input is sufficient only to supply the radiation losses, and another method is necessary to make the gas hotter. The method proposed has been called magnetic pumping, and the principle is illustrated in Fig. 8. The plasma is alternately compressed and

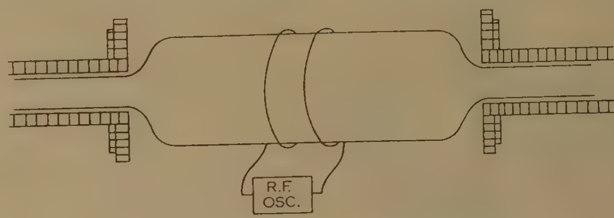


Fig. 8.—Schematic of magnetic pumping.

allowed to expand. If as much work is done on the magnetic field by the gas during expansion as was done by the magnetic field during compression, no heating would result, but under suitable conditions this is not the case. One way in which the plasma may be heated is by the pulsating field giving energy to the gyrating particles, which then distribute it between the three degrees of freedom by collisions. This makes the process irreversible.

(3.2.4) Effects of Impurities: the Diverter.

The Princeton group have also introduced a new method by which the residual impurities in a system may be reduced to a low value. It is to lead away an outer shell of magnetic flux from near the tube walls into a large auxiliary vacuum chamber where the lines are anchored to a metal baffle. The application of this idea is illustrated in Fig. 9, and the device has been

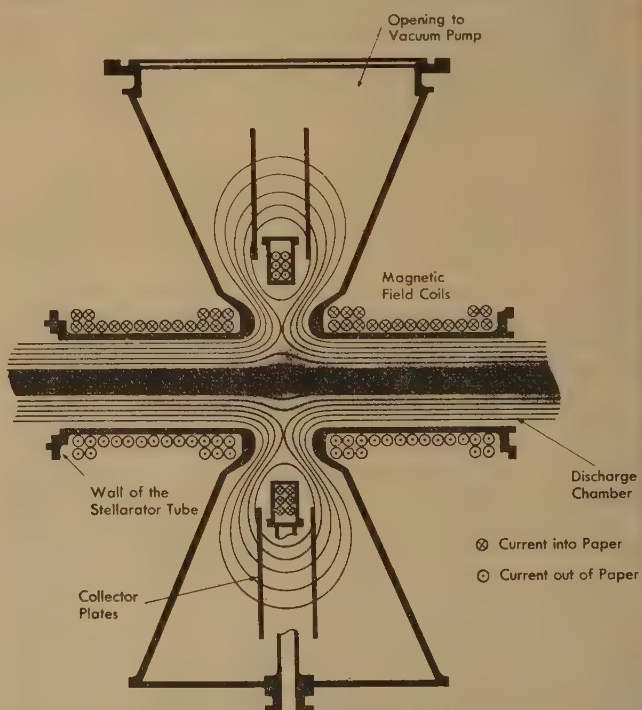


Fig. 9.—Arrangement of diverter.

called a 'diverter'.[27] Impurity atoms leaving the walls becom ionized, move along the magnetic flux lines and pass into th diverter, where they strike the baffle. Here they are neutralize and pumped away, instead of moving inwards and reaching th

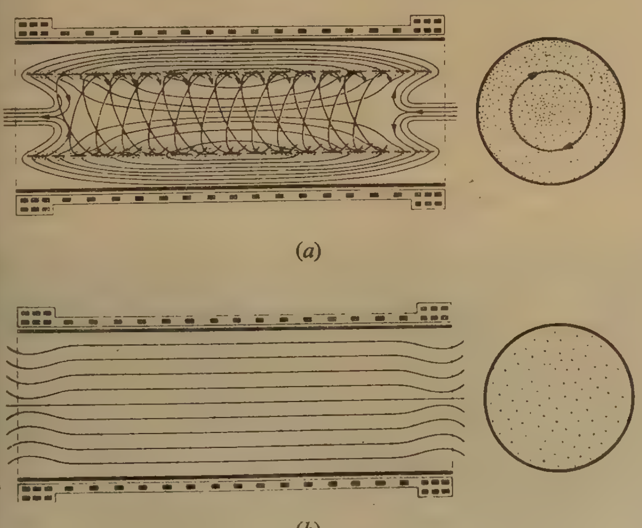
hottest parts of the discharge. Tests of the diverter system are reported to have shown a reduction of impurities in a helium discharge by a factor of 5.

(3.2.5) Present Status of the Stellarator Programme.

The devices of the Stellarator type which were described at Geneva are listed in Table 2. Most of the experimental work has been concerned with the initial ohmic-heating stage. The time taken for the plasma to reach the walls has been observed to be several orders of magnitude less than the theory predicts; moreover, this time, instead of being proportional to the square of the magnetic field, appears to be proportional to its square root. A large Stellarator installation known as model C is nearing completion at Princeton; cósting about £10 000 000, it is expected to begin operation early in 1960. 1 500 MJ of energy is stored in rotating machinery, 600 MJ being taken out in a time of the order 3 sec. This will be used to provide an axial magnetic field of 50 kG in the model C machine, which will be of 20 cm bore and of race-track form. The production of a field of this strength poses severe structural problems, since the bursting forces on the coil are equivalent to a pressure of about 100 atm. Helical windings will be used to provide the rotational transform. Various megajoule condenser banks supply the energy for heating the plasma.

(3.3) The Astron

Another fusion device using the principles of closed magnetic lines is the Astron,[28] proposed by Dr. Christofilos of the Livermore Laboratory. Here the pattern of lines is produced by the combination of the currents in an external solenoid and those in a cylindrical sheet of rotating relativistic electrons. The sequence of events is that first the external coil is energized, producing an almost uniform homogeneous axial field [Fig. 10(*a*)]. Then,



(*a*)

(*b*)

Fig. 10.—The Astron.
(*a*) Stage 1.
(*b*) Stage 3.

using a pulsed heavy-current source of high-energy electrons and travelling magnetic-wave injection, the rotating cylinder of electrons called the E-layer is built up. The current in the E-layer reduces the axial field inside it, and finally, when there are enough electrons circulating in the layer, the field inside is reversed and a pattern of closed lines is created [Fig. 10(*b*)]. Neutral fuel gas is now admitted to the system, is ionized and heated by the relativistic electrons, and forms a hot plasma within the field pattern which is further enhanced by the resulting diamagnetic currents in the plasma. During each of the successive phases it is necessary to adjust the current in the external coil to maintain the required conditions. Since the outer magnetic lines near the wall are not closed, diverter action for wall-emitted impurity atoms is automatically provided in the Astron.

The energy of the injected electrons in the layer is determined by stability and heating requirements and is estimated to be 50 MeV for a full-scale reactor. As a reactor the Astron is envisaged as a steady-state device with periodic replenishment of the E-layer and continuous injection of neutral fuel gas. Although a good deal of theoretical work has been done, there are still doubts about the stability of the E-layer with 1–3 MeV electrons. To date, only the electron gun has been built and tested; this gives 60 amp of 700 keV electrons in 0·5 microsec pulses with good collimation.

(4) OPEN-LINE CONTAINMENT SYSTEMS

(4.1) Magnetic Mirrors or Adiabatic Traps

Another approach to containment has been the use of an externally generated magnetic field having a field pattern of the form shown in Fig. 11, where the magnetic lines pass out of the vessel. The ionized particles will move along helical paths as described before. As they move into a region of greater field strength, their speed of rotation about the field lines increases. If the magnetic field increases sufficiently, the particle energy becomes entirely rotational and the particle then starts to drift back. The condition for trapping on this independent particle model is that

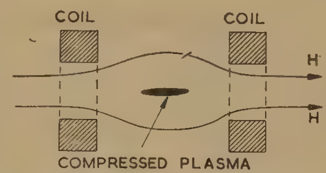


Fig. 11.—Schematic of mirror device.

$$\frac{W_{11}}{W_1} \leqslant \left(\frac{H_{max}}{H_{min}} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where W_{11} and W_1 are the components of the particle energy respectively parallel and perpendicular to the axis and H_{max}/H_{min} is the mirror ratio. For particles moving at small angles to the axis this condition may not be satisfied and these will be lost. This applies to particles having velocity vectors lying within a cone whose half-angle α is defined by

$$\sin\alpha = \sqrt{\frac{H_{min}}{H_{max}}} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

All particles with velocity vectors outside this angle would, in principle, be trapped. In practice, particles with initial velocity vectors in the trapping range might still be lost after making collisions, since their energies may be changed and their velocity vectors moved into the escape cone. For the energy released to balance the energy lost by fast particles escaping from the system, the ratio between the probability of the particles taking part in nuclear collisions and the probability of their making scattering collisions must be made as large as possible. Since the coulomb scattering cross-section is $\propto/T^2$, it is necessary to operate this type of system at temperatures about ten times higher than those required for systems using closed lines of containment. The need for these higher temperatures means in practice that the β-values for mirror machines should be reduced by an order of magnitude if a comparison is being made.

It is important to know whether high β-values will be stable for open-line systems, and to date there is little evidence on this.

Post *et al.*[29] have demonstrated that a β-value of about 0·08 is stable, but they have emphasized that neither the theoretical predictions nor experimental results are sufficiently far advanced to predict stable plasmas at the temperature and pressure necessary for a reactor. Trubnikov *et al.*[30] pointed out that radiation from electrons rotating in a magnetic field can be large at harmonics of the cyclotron frequency. This may be found to impose a serious limitation on high-β-value systems.

(4.2) Injection

The injection of plasmas into mirrors has received considerable attention, since it is a serious technical problem, the difficulty arising from the impracticability of trapping particles in a static system. In order that they shall be retained, some parameter must change after injection: use has been made of injection through time-varying r.f. fields, plasma injection into a rapidly rising mirror field, trapping due to collisional or co-operative particle reactions taking place after injecting a low-temperature plasma into a mirror, and change of charge or of mass after entry into the mirror.

(4.3) Experimental Installations and Results

Some of the characteristics of the machines of this type which were described at the Geneva Conference are listed in Table 2.

(4.3.1) The Livermore 'High-Compression' Experiment.

In one apparatus[29] a burst of low-energy plasma was injected into an evacuated chamber immersed in an initially weak magnetic mirror field. This field was then increased by a factor of 1 000 in a period of 200–500 microsec and the effects on the plasma were observed. The starting density of trapped plasma was in the region of 10^{11}–10^{12} ions/cm^3, and this was increased by compression to about 10^{14} ions/cm^3. Observations on the compressed plasma and the escaping particles showed that the half-value containment time was of the order 1 millisec, with energetic components remaining as long as 20 millisec. Electron energies were measured over a range of 3–120 keV. It was estimated that these measurements would have been consistent with a Maxwellian energy distribution corresponding to an electron temperature in the region of 10–20 keV, i.e. $1\text{–}2 \times 10^8$ deg K. Under the conditions used the ion mean energies could not have been as high as the indicated electron temperatures; with different conditions ion energies of 2–3 keV were observed.

In another apparatus the magnetic field was continuously increased during the injection of 10 keV deuterium ions. To minimize the loss of particles by charge exchange, injection should be into the highest attainable vacuum, the Livermore device being pumped down to pressures less than 10^{-9} mm Hg.

(4.3.2) Ogra.

Other high-energy ion-injection systems use a change in the mass ratio of the ions to trap them, the magnetic field remaining constant. This is illustrated in Fig. 12. Ogra[31] (Fig. 13), the very large Russian machine, is designed to utilize this principle; basically, this machine consists of a cylindrical vacuum chamber 12 m long and 1·4 m bore with external coils providing a main magnetic field of 5 kG gauss, increasing to 8 kG in the mirrors or traps at either end. A power input of 4 MW is required to maintain these fields. The chamber can be baked at 450° C for outgassing, and by the combination of mercury-diffusion and ion-sorption titanium pumps it is hoped that a final pressure of 10^{-8} mm Hg will be obtained. Molecular ions (H_2^+ or D_2^+) with energies up to 200 keV will be injected transversely into the machine in such a manner that if they remained unchanged they would travel a distance of 1 km before returning to the injector. It is hoped that their dissociation and consequent trapping in the machine will occur through collisions with the residual neutral gas. Balancing the favourable process of dissociation against the unfavourable one of charge exchange, it can be shown that the build-up of plasma in the machine will take place only if the injector current exceeds a certain critical value; it was claimed that for the Ogra machine this was 200 mA for H_2^+ ions.

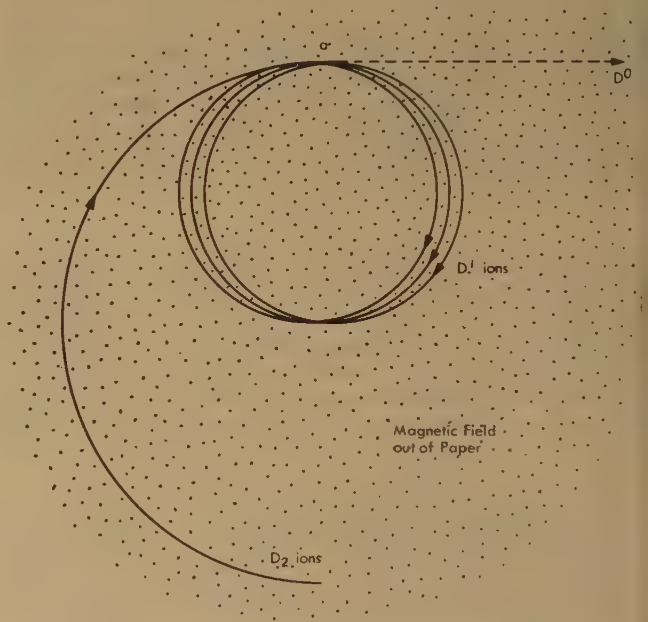


Fig. 12.—Break-up of molecular ion.

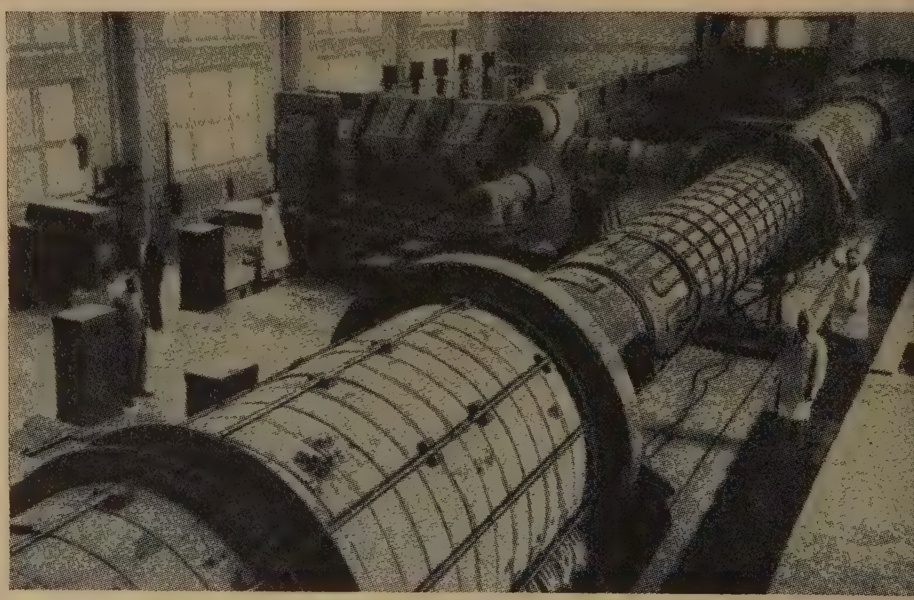


Fig. 13.—General view of Ogra.

(4.3.3) **D.C.X.**

The D.C.X. machine[32] built at Oak Ridge, Tennessee, also uses the principle of high-energy molecular-ion injection followed by dissociation. It differs from Ogra in that dissociation is achieved by passing the injected ions through a carbon-arc column which runs longitudinally through the machine (Fig. 14).

Fig. 14.—Discharge in the D.C.X.

The carbon electrodes themselves are in a subsidiary vacuum chamber, with the arc column running through baffles into the main chamber. Molecular ions passing through the column are dissociated with efficiencies of more than 70%. The main magnetic field is 10 kG, increasing to 20 kG in the mirrors at each end. Since the dissociation occurs in the arc column, it is unnecessary to provide for long molecular-ion trajectories, and hence D.C.X. is very much shorter and more compact than Ogra. As before, the competing processes of dissociation and charge exchange set limits on the injector current and background gas-pressure if a substantial plasma is to build up in the machine. At present, 300 mA of 600 keV molecular ions is injected and the background gas pressure is 10^{-7} mm Hg. It is estimated that an order of magnitude improvement in both the current and the vacuum is required before plasma formation can be achieved. With the existing apparatus the containment of single particles for 10 millisec has been achieved.

(4.3.4) **Scylla.**

Another system described was Scylla.[33] Built at Los Alamos, it consists of two single-turn coils mounted coaxially and used to shock-excite the plasma to 10–20 eV and then to heat it further by adiabatic compression. The coils are each of about 7 cm diameter and 2·5 cm wide, and are spaced with their centres 7 cm apart. Each coil was fed by a separate switch and condenser bank. The initial current in the coils produces a circumferential current in the gas near the tube wall. This is repelled inwards, setting up a shock wave and heating the gas. The continuing build-up of current in the coils then compresses the plasma in a mirror configuration. The mirror ratio is 1·4 and the maximum central field after 1·5 microsec is 38 kG. Neutrons have been observed; their correlation with the energies of the deuterons making the nuclear collisions do not show any marked anisotropy, but no claim has been made that a thermonuclear reaction has been established.

(5) MISCELLANEOUS SYSTEMS

Devices described at Geneva which come under this heading are listed in Table 2. In two of them the loss of particles along lines of magnetic field is prevented by causing the plasma to rotate. With a magnetic field of the mirror type, motion along the lines of magnetic force also involves motion towards the axis of symmetry, and this is opposed by a centrifugal force if the plasma itself is rotating. The two systems of this type described were by workers from the Berkeley (Homopolar)[34] and Los Alamos (Ixion)[14] laboratories. In both cases the rotation is achieved by applying a strong radial electric field between a central axial electrode and an outer ring (Fig. 15).

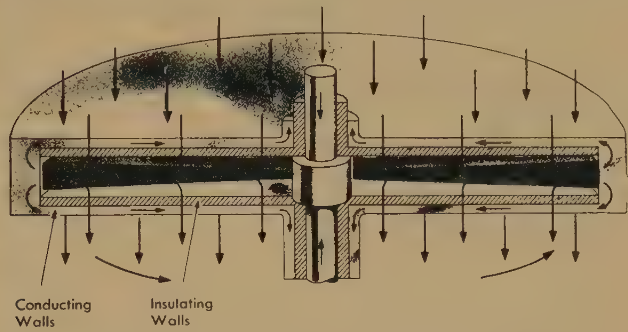


Fig. 15.—Cross-section of homopolar device.

The behaviour of the whirling plasma is analogous to that of a Faraday disc or homopolar motor. In the present experiments the rotation has been measured, using the Doppler shift of line radiation, and shown to agree with theory. Apart from the containment of hot plasma, these devices also have interesting possibilities for energy storage. In this respect they behave very much like capacitors, but because of their very high rotational speeds they can store energy at higher energy densities. Moreover, unlike conventional homopolar generators with material rotors, plasma homopolars can give up their energy extremely rapidly. The disadvantage is that, at present, possibly owing to electrode effects, high internal losses lead to self-discharge times of the order of milliseconds.

(6) CONCLUSIONS

I have outlined only the main approaches being followed in this search for thermonuclear power. Other workers have studied plasma containment by r.f. fields[35, 36] and devices called the ion magnetron[13] and the Osovetz[37] plasma loop. The ingenuity shown in all these systems has been considerable. The experimental results obtained indicate that the behaviour of fully ionized plasmas is more complex than can be accounted for by existing theories. With so many gaps shown up in our knowledge, it is too early to say that any one approach is likely to provide the best road towards the goal, and a great deal more work still remains to be done. This research is not confined to the field of physics, since engineering problems also require study. In general terms, the engineering problems are: (*a*) the economic storage of energy in the tens to hundreds of megajoules region; (*b*) its switching into and out of circuit, in some cases in very short times and with reactive loads; (*c*) the design of a magnetic-field system to withstand the forces associated with establishing the high fields required; and (*d*) the problems of circuit protection. I would expect progress with many of these interesting problems to be described at future meetings of The Institution.

(7) ACKNOWLEDGMENTS

This lecture gives a brief survey of work being carried out in several countries and by many workers. Many references are given to sources where more detailed information may be found

although the list of references is not exhaustive. In particular, I wish to acknowledge the help received from Dr. R. J. Bickerton in preparing the lecture, to thank the United States Atomic Energy Commission for permission to use Figs. 2, 4–12, 14 and 15, and the Academy of Sciences of the U.S.S.R. for permission to use Fig. 13.

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The Institution of Electrical Engineers
Paper No. 2805 U
Jan. 1959

DISCRIMINATION BETWEEN H.R.C. FUSES

By E. JACKS, Member.

(*The paper was first received* 16*th August, and in revised form* 22*nd October*, 1958. *It was published in January*, 1959, *and was read before the* UTILIZATION SECTION 12*th February, and the* MERSEY AND NORTH WALES CENTRE 16*th March*, 1959.)

SUMMARY

The paper reviews the factors upon which discrimination between h.r.c. fuses depends and relates these to current practice.

Consideration is given to the fundamental functions in fuse operation and the principles of system planning to facilitate discriminative protection. Suggestions are also made concerning the presentation of data to assist in the choice of fuses.

Finally, brief examples are given of practical cases to illustrate the conception of degree in discriminative protection.

TERMS AND DEFINITIONS

Definitions are generally as laid down in B.S. 88: 1952 and as illustrated in Fig. 1.

For the purposes of the paper the term 'h.r.c. fuse' may be taken to refer to those fuses having rupturing capacities of the order of AC4, DC4 (i.e. 33 kA r.m.s. symmetrical) or above. The abbreviation h.r.c. (high rupturing capacity) is used in the paper because of its wide acceptance over many years, although by implication in B.S. 88: 1952 the term 'high breaking capacity' is preferred.

(1) INTRODUCTION

Discrimination between h.r.c. fuses has been achieved with practical success since these devices were first introduced. Progress in this respect has been mainly empirical and has been taken for granted by the majority of users. During the last decade the increased use of h.r.c. fuses has given rise to an increased interest in the theory of fuse operation. New knowledge has emerged to confirm or modify previous theories in the light of experience, and a wider appreciation of their capabilities has opened up new opportunities of extending their usefulness.

The purpose of the paper is twofold: first, to define the capabilities and limits of the modern h.r.c. fuse as regards discriminative protection and to indicate the extent of its further usefulness; secondly, to relate the theory of fuse performance to standard distribution practice. Although the treatment of the subject is kept within practical limits in relation to standard practice, it is recognized that special applications occur which require more detailed treatment and where the approximations permitted in standard practice would not be appropriate. The principles on which discrimination depend, however, are the same for all cases.

The subject is topical in as much as many users who have had practical experience of fuses over many years are now taking an interest in the more technological aspects of fuse performance and are interested in knowing how the new knowledge can be reconciled with established practice.

The key note in the achievement of the h.r.c. fuse as regards discrimination is its basic simplicity of construction, which permits consistency in manufacture and accuracy of characteristics. This does not mean that there is no subtlety in design, nor does it imply that the duty of the fuse is simple—on the contrary, fuse operation involves all the technology implied in circuit interruption. The theory of fuse discrimination must ultimately be based upon an appreciation of fuse operation under various short-circuit conditions. It must be realized that the principal duty of the fuse is to interrupt short-circuits and that discrimination is usually regarded as a secondary requirement. Nevertheless, a satisfactory compromise between the two can be achieved.

From a consideration of the technological factors, discrimination depends on the correct application of the fuse in any given situation. Practical and economic considerations external to the fuse itself can sometimes significantly affect the results obtained.

It should be emphasized that the remarks and opinions expressed in the paper can be taken to apply only to the modern h.r.c. fuse manufactured to high standards.

(2) OBJECT OF DISCRIMINATION AND THE BASIC FUNCTIONS INVOLVED

(2.1) Definition

Discrimination may be defined as the ability of protective devices to interrupt the supply to a faulty circuit without interfering in any way with the source of supply or the remaining healthy circuits fed from it. This necessarily implies co-ordination between two interdependent devices in series.

(2.2) System Requirements

The requirements for achieving discrimination and the degree to which it is required will vary with the nature of the system in question. In some systems discrimination predominates, e.g. certain transmission systems require specially matched and calibrated relays to achieve a close ratio between the time/current settings. In other systems, and particularly in medium-voltage systems such as supply networks or industrial installations where h.r.c. fuses are normally used, such a high degree of sensitivity is seldom necessary. These systems normally branch out from distribution points or busbars into sub-circuits such that the distributor fuse can more often than not be several times the current rating of the sub-circuit fuse. The ratio occurring between the two is usually of the order of 2 : 1 or more, and such applications account for the largest proportion of locations in which h.r.c. fuses are employed.

The h.r.c. fuse can, in fact, offer a greater degree of discrimination than 2 : 1 under certain conditions, and is often required to do so; but this does not alter the fact that the best engineering practice is to arrange the layout of the system to afford as great a margin as possible. Where a greater degree is required, a knowledge of the circuit conditions and fuse characteristics are necessary in order to make a correct choice. Whatever degree may be required, it follows that any protective device must be capable of giving consistent protection for the whole of its life in service.

Mr. Jacks is with the English Electric Co., Ltd.

(2.3) Factors affecting Discrimination

(2.3.1) Fuse Performance.

H.R.C. fuses do not give close discrimination at very high short-circuit currents because of their properties of current and energy limitation which are the more important factors, but will do so at the lower values of over-current. The performance of an h.r.c. fuse designed to have the property of non-deterioration can be predicted for any given circumstances, because it remains faithful to its declared characteristics. The data required for fuse discrimination can be made readily available (see Sections 3.9–3.11).

(2.3.2) System Planning and Layout.

Discrimination can depend as much upon the design of the system as upon the performance of the protective devices involved, and the best results can be achieved only if the protective devices are co-ordinated with the system as a whole in the first stage of planning. An attempt to plan a distribution system merely on the basis of load requirements, with protective devices as an afterthought, will usually result in unnecessary complication and defeat its own object.

(2.3.3) Non-Deterioration.

All forms of protection, whether discriminative or otherwise, depend for their effectiveness upon the manner in which their pristine condition can be preserved in service. They must either possess the property of complete non-deterioration or must be retested, maintained and recalibrated at regular intervals throughout their service lives. Since fuses are static devices without moving parts, they are not expected by the user to require maintenance and are seldom disturbed over periods of years unless a fault occurs. Non-deterioration is therefore one of the essential properties on which discrimination depends.

(3) FUSE CHARACTERISTICS AFFECTING DISCRIMINATION

(3.1) Fuse Operation

Positive discrimination between any two h.r.c. fuses in series is achieved when the larger or 'major' fuse remains unaffected by fault currents which cause the smaller or 'minor' fuse to operate (or blow). When an h.r.c. fuse operates, the element absorbs energy from the circuit and heats up until it melts, vaporizes and disperses. This action is then followed by a period of arcing which persists until the resistance across the fuse builds up to a sufficiently high value to reduce the current to zero. The former period is known as the pre-arcing period and the latter as the arcing period (see Fig. 1). The heat produced within the fuse-link during operation is the integral $\int i^2 r dt$, where i is the instantaneous current and r the instantaneous resistance during the operating time.

(3.2) Effect of Fuse Resistance

The resistance of two fuse-links chosen to discriminate with one another will be different, and each will change as the elements heat up during the pre-arcing period. The rates of change of r (i.e. dr/dt) during this time will also be different, but the rate of change in the minor fuse will be greater than that in the major fuse, so tending to assist discrimination.

The resistances of the fuses in relation to the resistance of the rest of the fault circuit are not usually very large. Up to the point where the minor fuse begins to arc, the change in the resistance of either fuse will not therefore affect the current appreciably. This is obvious from the oscillogram in Fig. 1,

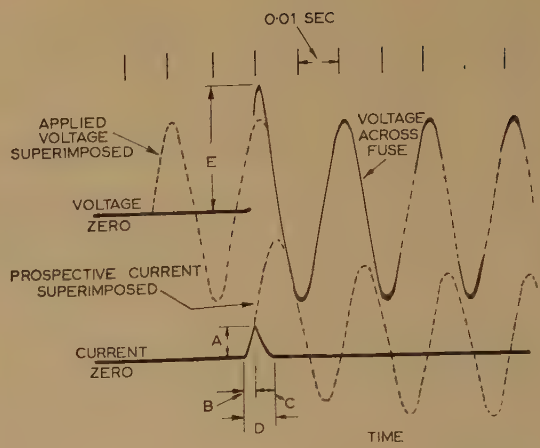

Fig. 1.—Typical oscillogram of fuse operation on short-circuit, showing conventional terms as defined in B.S. 88: 1952.

Test Circuit Conditions

Prospective currents:	
r.m.s. symmetrical	47·6 kA
r.m.s. asymmetrical	72·5 kA
peak asymmetrical	102·5 kA
Applied voltage	440 volts
Power factor	0·13
Frequency	50 c/s

Fuse Performance

A. Cut-off current	29·2 kA
B. Pre-arcing time	0·002 25 sec
C. Arcing time	0·004 35 sec
D. Total operating time	0·006 6 sec
E. Maximum arc voltage	856 volts

which shows that the voltage drop across the fuse is very small during the pre-arcing period. Fuse resistance is a factor in discrimination, but only to a small degree, and since changes in resistance tend to assist rather than complicate the desired effect, it is convenient to ignore them for all practical purposes and to consider only I and t in calculations.

(3.3) Conception of I^2t

The choice of major and minor fuses can then be made on the basis of I^2t (amp^2-sec), because the same current flows through both fuses for the same time and I^2t is the factor common to both. Values* of I^2t for either the pre-arcing or arcing period (or for the total operating period, which is the sum of both of these) can be obtained from test oscillograms or by other means.

The melting of a fuse element is the point of no return. Even if the circuit is interrupted elsewhere, once the element has liquefied it cannot be restored to its former state or even become a stable conductor. Thus, for the purpose of discrimination the element of the major fuse must not approach this point. In other words, the total I^2t (i.e. pre-arcing I^2t + arcing I^2t) admitted during the operation of the minor fuse must not exceed the pre-arcing I^2t of the major fuse. There must, in fact, be sufficient margin between the two that the major fuse element is not permanently affected by the I^2t which has to pass through it to blow the minor fuse. This principle is exemplified in Fig. 2, which shows I^2t values extended from test oscillograms of typical major and minor fuses.

(3.4) The Pre-Arcing Period

It is well known that h.r.c. fuses exhibit the property of cut-off, and at prospective currents at which cut-off occurs for a given fuse, the pre-arcing period is of the order of $\frac{1}{4}$ cycle. (Fig. 3 shows typical values taken from tests on an existing range

* I is the r.m.s. current during fuse operation and is deduced from $\int i^2 dt$ over the pre-arcing, arcing or total operating time as appropriate.

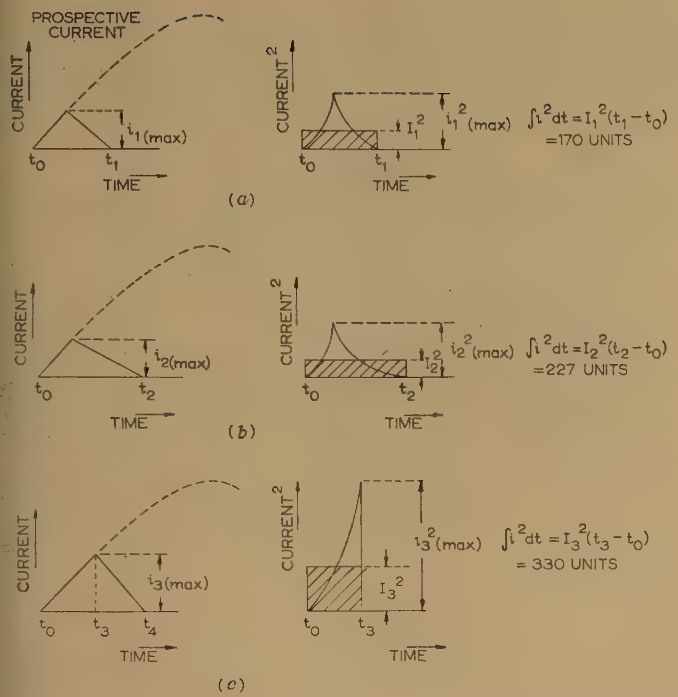

Fig. 2.—Method by which $\int i^2 dt$ values may be extended from oscillograms of typical major and minor fuses.

(a) Minor fuse having an arcing time similar to its pre-arcing time.
(b) Minor fuse having an arcing time of twice its pre-arcing time.
(c) Major fuse.

I_1 = (r.m.s. value of $i_1)_{t_0-t_1}$.
I_2 = (r.m.s. value of $i_2)_{t_0-t_2}$.
I_3 = (r.m.s. value of $i_3)_{t_0-t_3}$.

Since the total $\int i^2 dt$ of either minor fuse is less than the pre-arcing $\int i^2 dt$ of the major fuse, discrimination will be obtained.

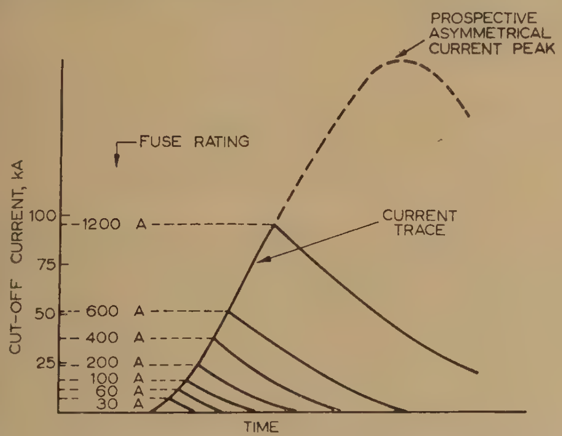

Fig. 3.—Cut-off currents, for a given prospective current, for a family of fuses.

Prospective current:	
r.m.s. symmetrical	80 kA
r.m.s. asymmetrical	100 kA
peak asymmetrical	177 kA
Applied voltage	600 volts
Power factor	0·15
Frequency	50 c/s

of fuses.) For such short times it may be assumed that all the pre-arcing energy is absorbed by the element itself without loss to adjacent material and, being directly proportional to the mass of the element, is sensibly constant. The pre-arcing energies recorded empirically are lower than those calculated from the known physical constants of a particular element, owing to the effect of electromagnetic forces which cause the softened element to change form in the last stage of melting. At relatively low prospective currents, involving absence of cut-off and longer pre-arcing times, heat is lost from the element and the pre-arcing energy is consequently no longer constant but increases as a function of time. Fig. 4 shows the relationship between the

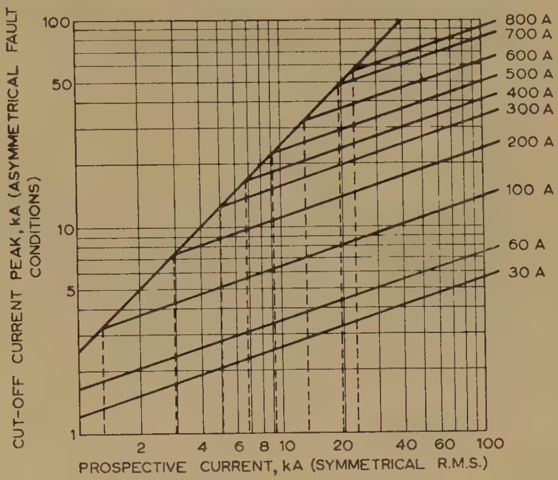

Fig. 4.—Relationship between prospective and cut-off currents for a family of fuses, and the points at which cut-off commences.

prospective current and cut-off for a family of fuses referred to in Fig. 3 and indicates the prospective currents at which cut-off commences for each fuse.

(3.5) The Arcing Period

The energy dissipated in the fuse during arcing consists of two components: one is derived from the inductive, or stored, energy of the circuit and the other is the energy fed into the arc directly from the source of supply.

The stored energy, $\frac{1}{2}Li^2$, is a function of the circuit inductance and the peak instantaneous current—which, in a fuse interrupting a heavy short-circuit, is the cut-off current. The inductance will differ for each circuit, as expressed in the power factor of an a.c. circuit or the time-constant of a d.c. circuit. The energy derived from the source of supply is a function of the instantaneous system voltage during the arcing period. For d.c. systems this is practically constant, but for a.c. systems it may vary from zero to peak and depends upon the point on the wave at which arcing commences.

The energy from both sources must be dissipated as heat within the arc. For given circuit conditions the design of the fuse can be varied to absorb the heat (and so cool the arc) at a desired rate. In the initial stages of arcing this can be achieved by varying the characteristics of the arc itself, and in the latter stages by varying the cooling medium. It must be assumed that the fuse has been designed and rated to keep the arc energy to a minimum consistent with the ability of the fuse to absorb the energy. In this connection it is important to ensure that the applied or system voltage does not exceed the rated voltage of the fuse.*

The circuit constants have as great an influence on the arcing energy as the characteristics of the fuse itself, and it will be appreciated that the variables introduced from both sources

* If a well-designed fuse is used on system voltages which are within its voltage rating, the current decrement during arcing will be reasonably uniform. It can be shown that, where the pre-arcing and arcing currents follow uniform rates of rise and decrement, the arcing I^2t of a minor fuse may be twice that of the pre-arcing I^2t and still discriminate with a major fuse with double the rating of the minor fuse (see Fig. 2).

make the accurate prediction of arc energy a difficult problem. For a given fuse, however, it is possible to determine a maximum value of arcing I^2t which can occur. This is done in practice by calculating those circuit constants which will give the most severe arcing conditions and confirming from measurements taken from actual tests. For a.c. conditions the test involves applying the short-circuit at a point on the voltage wave which produces appreciable asymmetry of the current and at a critical value of prospective current (which is not necessarily the highest).

(3.6) Short-Time Operation

When a minor fuse exhibits cut-off during the interruption of a heavy fault, the I^2t admitted during the arcing period is appreciable compared with that admitted during the pre-arcing period. In most cases the former exceeds the latter and is therefore a substantial part of the total operating I^2t.

It would be impracticable to take into account all the varying values of arcing I^2t when making a choice of minor and major fuses for the purpose of discrimination. The most convenient method and the safest is to assume the worst conditions and to consider only the maximum value, as explained in Section 3.5. If discrimination is possible at this maximum value, it follows that it will be obtained at the lower values which occur on the less-severe circuit conditions. It is important to remember that the maximum value would occur only under the most unusual conditions and would seldom be realized in service. The use of the maximum value therefore provides a useful margin of contingency.

(3.7) Long-Time Operation

At the lower fault currents at which cut-off does not occur the arcing I^2t is small compared with the pre-arcing I^2t. As previously mentioned, the pre-arcing energy increases because of the increased thermal loss from the element, and at the same time the arcing energy is lower because the inductive energy in the circuit is less. Furthermore, under a.c. conditions the arc will tend to extinguish at voltage zeros and possibly at the next voltage zero after arcing commences. Thus, for operating times longer than approximately one cycle (0·02 sec at standard frequency) the arcing energy may be almost negligible, as illustrated in Figs. 5(*a*) and 5(*b*).

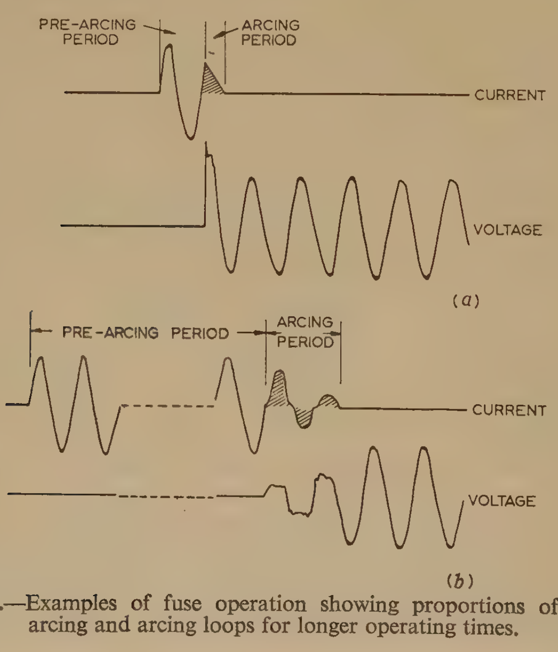

Fig. 5.—Examples of fuse operation showing proportions of pre-arcing and arcing loops for longer operating times.

Since the arcing I^2t is small compared with the pre-arcing I^2t, it can be ignored for practical purposes and discrimination can be judged by merely comparing the pre-arcing I^2t of the minor fuse with that of the major fuse. This is a simple matter of direct reference to the published time/current characteristics.

(3.8) Two Zones of Operation

It is, then, apparent that the factors of h.r.c. fuse discrimination resolve themselves under two distinct headings:

(*a*) Those involving low prospective currents which produce pre-arcing times longer than approximately 0·02 sec.
(*b*) Those involving high prospective currents which produce pre-arcing times shorter than 0·02 sec.

The choice of fuses is made in terms of current rating. This is a convention which is necessary because the first consideration in choosing fuses is to ensure that they are adequate to carry load under healthy circuit conditions. It does not always follow that the cut-off characteristics and I^2t values of a range of fuses conform to a regular relationship with respect to current rating. It is thus necessary to set down time/current and I^2t characteristics for each fuse in order to translate these data into terms of current rating.

(3.9) Time/Current Curves

Fig. 6 shows typical time/current characteristics in which prospective current is plotted against pre-arcing time as pre-

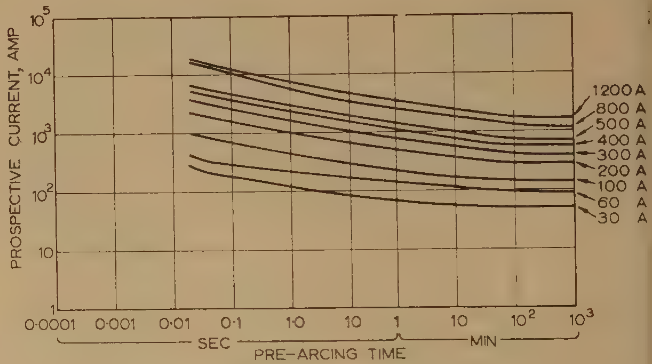

Fig. 6.—Time/current characteristics.

scribed in B.S. 88: 1952. For pre-arcing times longer than 0·02 sec it has been shown that the prospective current and the current admitted by the fuse are similar; thus, for a given current, discrimination depends only on the pre-arcing time and can be obtained between successive steps of current rating. For example, in a circuit where the prospective current is 3 kA the pre-arcing time of a 300 amp fuse exceeds 0·02 sec and will discriminate with the next rating in the range, namely 350 amp. The 350 amp rating will discriminate with a 400 amp one, and so on.

(3.10) Values of I^2t

Fig. 7 gives values of I^2t corresponding to each fuse rating and plotted in convenient form; from it the total $(I^2t)_{max}$ for any minor fuse can be determined. Then, by extending this value to the curve representing pre-arcing I^2t values, the nominal rating of the smallest major fuse which will give positive discrimination can be read. The practical choice will then be the next larger standard rating. For example, the rating of the minor fuse is known to be 100 amp in a circuit having a prospective current of 5 kA [which is above the value at which this fuse begins to cut off (see Fig. 4)]. Fig. 7 shows the total I^2t of

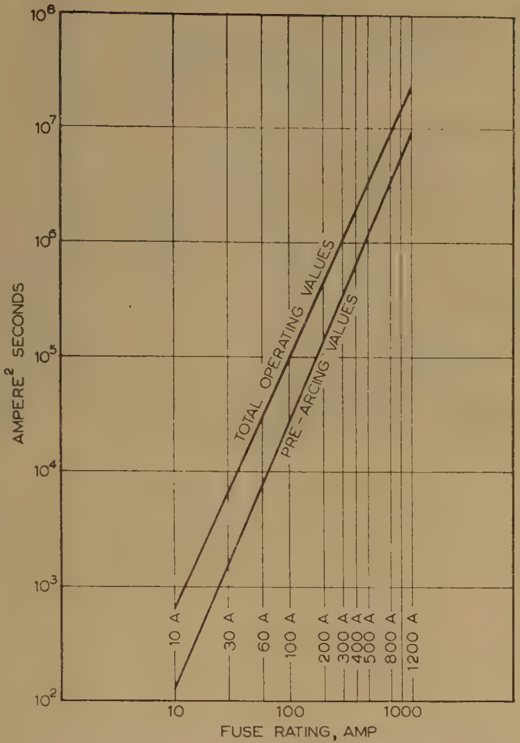

Fig. 7.—$\int I^2dt$ characteristics showing relationship between total and pre-arcing $\int i^2dt$ values for a family of fuses.

tive current would have to flow in a fuse to produce the same quantity of energy as would be produced if the actual current during the period of operation considered flowed in the fuse for the actual period'.

This time t_v is calculated from

$$t_v = \int i^2dt/I_p^2$$

in which I_p is the prospective current and t_v is the virtual time of the period of operation under consideration.

Fig. 8 shows virtual-time/prospective-current curves for a typical range of fuse-links. These require an accurate measurement of I^2t from test oscillograms in the same manner as for Fig. 7.

To choose fuses for the purpose of discrimination, virtual-time/current curves are required for both the pre-arcing and total operating I^2t values of each fuse-link. The virtual total operating time of the minor fuse can then be compared directly with the virtual pre-arcing time of the major fuse after a reasonable tolerance margin has been allowed between the two. The theoretical merits of the virtual-time concept are somewhat overshadowed by the practical difficulty of presenting the curves in an easily usable form.

(3.12) Consistency of Manufacture and Quality

The theories so far expounded have been rationalized to some extent in order to meet practical conditions. The data on which the theories can be applied in practice are derived from type tests on actual fuses and on the assumption that the fuses used in service will be identical in characteristics with those used in the type tests. This in turn implies that the fuses used in service

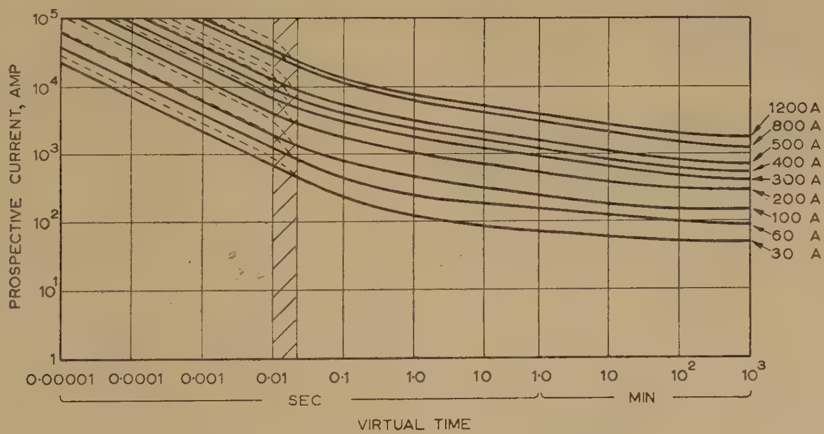

Fig. 8.—Virtual-time/current characteristics.

——— Characteristics for virtual pre-arcing times.
– – – – Virtual total operating times.

For times longer than 0·01 sec the total operating curves tend to the pre-arcing curves and may be interpolated as shown in the shaded region.

the 100 amp fuse-link to be 105×10^3 amp^2-sec. The standard fuse link whose pre-arcing I^2t exceeds this is a 200 amp unit with a pre-arcing I^2t of 150×10^3 amp^2-sec.*

(3.11) Virtual Time

An alternative to the above method of determining discrimination is suggested in recent British Standards. This is based on the concept of 'virtual time', which is defined in B.S. 2692: 1956 as 'the time for which a steady current equal to the prospec-

* In the case of the fuses taken to illustrate the graphical method shown in Fig. 7 the pre-arcing values were first made to conform to a straight line by adjusting the scale of the ratings. It was then found that $(I^2t)_{max}$ values taken from actual oscillograms also conformed approximately to a straight line. This indicates that for these particular fuses the relationship between pre-arcing I^2t and total operating $(I^2t)_{max}$ follows a rational law.

must be manufactured to a high standard of consistency. The accuracy of discrimination will therefore depend upon, and be proportional to, the consistency achieved. Consistency of characteristic is a function of quality control, but this in turn is dependent on design in as much as complication of design normally leads to difficulty in manufacture. Simplicity of design remains the keynote of consistency.

(3.13) Manufacturing Tolerances

Time/current characteristics are lines representing the mean value of tolerance bands, the width of which are a measure of the accuracy of manufacture. H.R.C. fuses can, with proper care, be made to conform to tolerance bands which are well

within the requirements of normal service duty, but it is incumbent on the user to know the tolerance band when choosing fuses to give discrimination.

(3.14) Non-Deterioration During Short-Circuit Operation

When a major and minor fuse are in series and the minor fuse blows, the major fuse must be able to handle the through short-circuit current without suffering damage. It will of necessity warm up, but provided that it cools down and resumes its pristine condition, it will then be able to discriminate in the future with a new replacement minor fuse. The stability of fuse characteristics under through short-circuit conditions is an essential factor and can be achieved by careful design.

(3.15) Proving Tests

In view of the many factors inherent in the fuse itself which affect discrimination and the varying incidence of such factors in service, the only way to prove the ability of a particular range of fuses to discriminate with one another is by empirical means. The tests should involve various combinations of major and minor fuse-current ratings, each of which is tested at various prospective currents from minimum current up to the rupturing-capacity rating. Other tests should take the form of consistency checks on stock fuses and careful inspection of unblown major fuses to check for non-deterioration. All tests require to be done on a sufficiently large scale to cover all possible variations.

(4) FACTORS OTHER THAN FUSE CHARACTERISTICS AFFECTING DISCRIMINATION

There are several factors which affect discrimination other than those which are inherent in the characteristics of the h.r.c. fuses themselves, but they are not particularly critical and can be accounted for in system planning.

(4.1) Unequal Loading

Fig. 9 shows a hypothetical branching circuit in which the main fuse feeds a busbar from which are fed four sub-circuits.

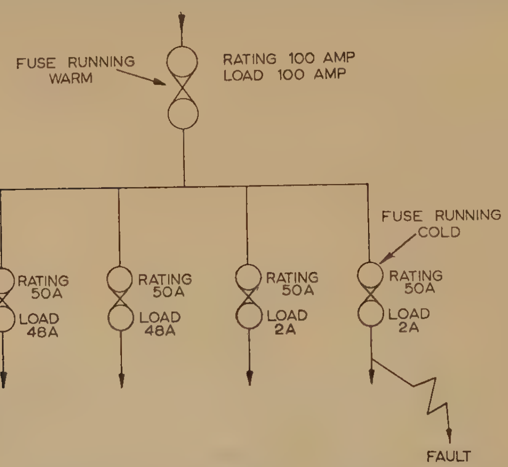

Fig. 9.—Example of unequal loading which might affect discrimination.

The aggregate of the current ratings of the minor fuses exceeds that of the major fuse by a considerable amount, but the aggregate of the loads on the minor fuses just about equals the full load on the major fuse. The circuit shown is exaggerated to illustrate the point and is unlikely to occur in this form in practice, but the tendency towards unequal loading is not uncommon and it is of interest to consider its effect. It can be surmised that the minor fuse, carrying the light load of 2 amp (in Fig. 9), will be running almost at ambient temperature, whereas the major fuse, carrying full load, will be running warm. If, then, a fault occurs on the load side of the lightly loaded minor fuse, a discrimination ratio as decided from the fuse characteristics will be affected, particularly for the longer operating times. Such cases do not occur very often in practice and could usually be avoided by proper layout of the installation. Even where they do occur there are several mitigating factors, because the difference in running temperature between the two fuses is not as significant as may be imagined. A proper evaluation of the difference can be made only by considering the running temperature of the elements within the two fuses in relation to the melting temperature of the metal from which those elements are made. For properly designed fuses in which the elements are proportioned to run relatively cool, the problem of unequal loading therefore becomes a minor factor.

(4.2) Effect of System Layout

Fig. 10(*a*) shows another circuit layout which occurs quite commonly in industrial systems and is contrary to the best

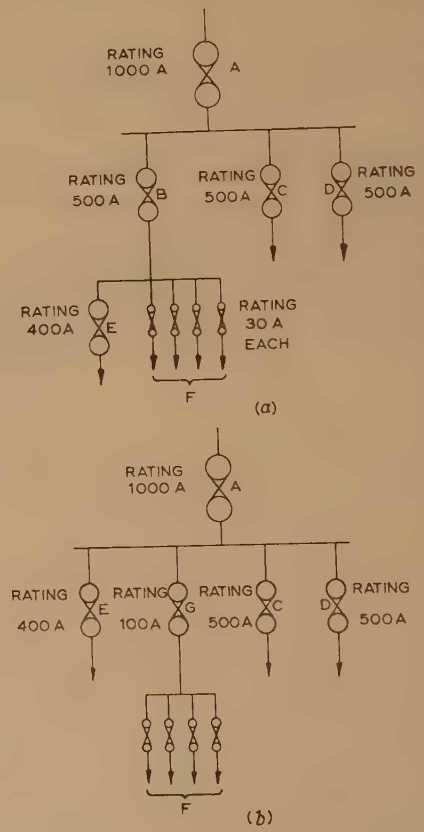

Fig. 10.—Effect of system layout on discrimination.
(*a*) B, C and D will discriminate with A, but E will not discriminate with B.
(*b*) G is a new fuse. All fuses will now discriminate.

practice where a high degree of fuse discrimination is either desirable or necessary. If it is assumed that for given circumstances in this particular system a ratio of 2 : 1 between the major and minor fuse is necessary to give discrimination, the sub-sub-circuit fuse of 400 amp may not discriminate satisfactorily at the highest fault current with the distributor fuse of 500 amp. The remedy here is to feed the 400 amp circuit direct from the busbars, as shown in Fig. 10(*b*). The original sub

circuit distributor can then be reduced from 500 amp to a size appropriate to the diversity on the remaining sub-sub-circuits. The economics of the change will vary according to site conditions, but should be easily assessable.

(4.3) Prevention of Abuse

The planning of all schemes of protection must proceed on the assumption that the equipment specified and installed is not unduly abused. When considering fuses this assumption may be safely made, because there is so little which can go wrong. Even fuses, however, must be installed and connected by electricians with sufficient skill to realize the importance of good connections.

H.R.C. fuse-links are normally tested in the fuse handles and fittings in which they are designed to be used in service. It is not uncommon, however, for fuse-links to be purchased and applied in equipment other than that for which they were originally designed. It is therefore a prerequisite of such applications that the terminals to which the fuse-links are attached should be adequate and similar in all essential respects to those which the fuse-link designer had in mind. It must be remembered that the link is a thermal device and must export some of the heat generated within itself to the contacts to which it is attached. Fuse fittings are normally designed on this basis, so that the link runs within the temperature limits required to ensure non-deterioration. It is prudent for the manufacturer to design fuse-links to carry their full load currents at temperatures well within the permitted limits, so as to allow an adequate margin for contingencies of service. If, however, there is blatant disregard of normal precautions, e.g. the use of extremely small terminals and undersized cables, it is only to be expected that the running temperatures will be other than normal and discrimination may suffer to some degree. It is hardly necessary to mention that the same trouble will be experienced if the connections are badly made, because these in themselves will be a source of heat which will cumulatively worsen until the point is reached where the fuse may blow prematurely.

(5) DEGREES OF DISCRIMINATION

It has been shown that the ultimate degree of discrimination which can be expected from fuses can be accurately defined by considering the characteristics of the fuses themselves and arranging the layout of the system with due regard for these characteristics. It has to be accepted that a rule-of-thumb ratio of the order 2 : 1 in current rating is required between the minor and major fuse for a relatively high degree of discrimination in systems where the fault level is high. For systems where the prospective currents are low and where the fuses do not exhibit cut-off, the ratio will not need to be so wide. On the other hand, for extremely important or vital circuits the ratio might need to be increased beyond 2 : 1. The point has been made that these ratios do not give any practical difficulty in system layout. Neither do they impose economic restrictions, provided that they are considered at an early stage in the system planning.

The importance of discrimination in relation to other factors in a particular system or installation may vary. Sometimes it is necessary to forgo some of the benefits of discrimination and to recognize the degree which is required or which can be obtained by a given choice of fuses.

(5.1) Assessment of Degree

The degree of discrimination required for any particular system is influenced by a number of factors, but these are not usually critical so far as the choice of fuses is concerned. It will therefore suffice to consider three possibilities:

(*a*) Those cases where the fuses protect vital circuits in which the continuity of supply and minimization of fault damage is essential.

(*b*) Less vital circuits where the consequences of loss of supply are not of first importance provided that the shut down is not prolonged.

(*c*) Cases where fuses are required mainly as back-up protection in independent circuits and where discrimination may not be the first consideration, provided that the fault is cleared without damage to equipment or the source of supply.

(5.2) Vital Circuits

Among the cases coming within the first category are such vital circuits as power-station auxiliary supplies, power supplies to continuous-process plants and circuits controlling vital services. In such cases the economy in capital expenditure is secondary to the requirements of safety or maintenance of the supply or both, and there can be no compromise in the choice of fuses or the layout of the system. The wise thing to do is to allow as wide a margin between major and minor fuses as the system will permit, and then to ensure that this is not less than that which the system requires. In this way discrimination can be assured with a sufficient margin of safety to account for all contingencies, including the human element.

(5.3) Practical Solutions

Cases within the second category are becoming less easy to justify as industry becomes more dependent on electricity as a vital service. They usually occur through the rapid extension of existing systems which have not been planned to cater for such exigencies and where the cost of correcting the initial mistake makes it expedient to forgo the ultimate benefits of discrimination. There are those cases which occur by default because they are brought into being by practical electricians who are not familiar with the factors governing discriminative protection. The assessment of these cases is sometimes necessary after the event.

In such cases the degree of discrimination required will depend entirely upon economic factors. Installations in some industries are more fault-prone than others. Moreover, the speed with which the supply can be restored may vary according to the maintenance facilities available, and this will reflect on the losses sustained in production output or its equivalent. It must be appreciated that in all cases the lower degrees of discrimination can be tolerated if they result only in nuisance and do not involve danger.

In some instances discrimination may be forgone with proper discretion, as shown in the practical example given in Fig. 11. This illustrates an overhead busbar system which feeds a large number of small motors and is then looped off to a relatively large machine which is used only occasionally. There is no reason why the overhead busbar should not be used as the main distributor to such a motor; but if the fuses in the motor circuit are comparable with the main fuses, discrimination cannot be expected in the event of a heavy short-circuit in the region of the terminal of the large motor. Whether the consequences of this are important can be judged only in the individual case, but, since the consequences result only in inconvenience and do not jeopardize the safety of either personnel or property, the risk is easily assessed on technical considerations alone.

(5.4) Lighting Circuits

The lighting circuits in the same factory would be quite a different matter. Here, the consequences of general failure to discriminate might involve considerable danger to personnel if the factory were plunged into darkness while the machines were running. There is, however, no reason why the correct discrimination ratio should not easily be achieved for lighting

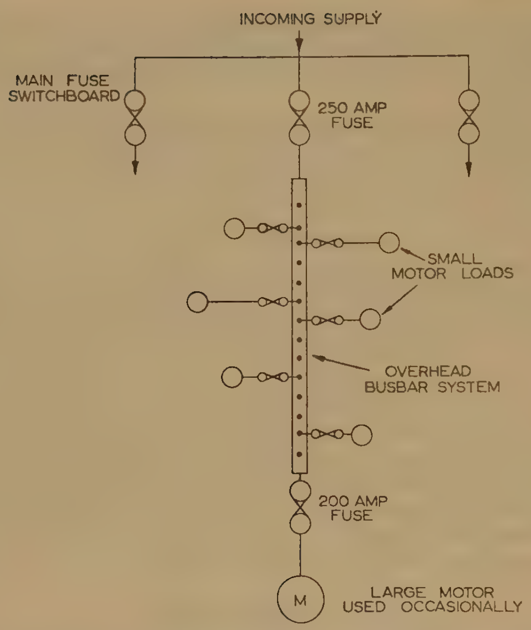

Fig. 11.—Example of system in which a low degree of discrimination may be expedient.

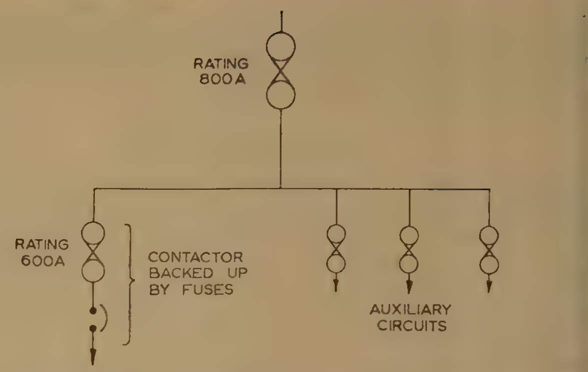

Fig. 12.—Example of arrangement in which the requirements of back-up protection predominate over discrimination.

circuits, because they consist mainly of small units of load evenly spaced and admirably suited for connection through branching sub-circuits of approximately equal size. Lighting circuits which do not discriminate are usually badly planned.

(5.5) Cases where Discrimination is a Secondary Factor

The third category covers only those specialized jobs where complete discrimination is known to be either inherently difficult or unimportant. Fig. 12 shows a typical instance. In this scheme the major fuse is required for clearing faults within the busbar zone. The size of the minor fuse is determined by the load of the particular sub-circuit which it controls.

Where such a scheme is adopted from expediency it is recognized that there is a band of fault currents at which discrimination cannot be expected. The minor 600 amp fuse is necessary because it affords back-up protection within the through short-circuit capacity of the contactor with which it is associated, whereas the major 800 amp fuse would not. Discrimination will, however, be obtained at the lower prospective fault currents, and because both fuses are relatively large, it will occur up to fault values of 15–20 kA (see Fig. 4).

(6) CONCLUSIONS

H.R.C. fuses provide discriminative protection to a degree of sensitivity which is well within the requirements of normal distribution. Their characteristics can be ascertained and declared accurately, so that they can be applied in any circumstances where the requirements are known. A practical method of presenting the necessary data is by curves similar to Figs. 6 and 7, which give direct values and avoid the abstract complications of alternative methods.

H.R.C. fuses remain consistent and stable in service without calibration or maintenance, provided that they are properly designed and manufactured. The property of non-deterioration is one of the most significant factors in accuracy of discrimination.

The design of the system in which fuses are used and other factors external to the fuse itself affect the degree of discrimination which can be achieved. Assessment of these factors is necessary for the most effective use of fuses, but the assessment need not be critical.

Failure to achieve discrimination owing to incorrect choice of fuses may result in nuisance, but need not involve danger if both the major and minor devices are h.r.c. fuses of proper design and manufacture.

DISCUSSION BEFORE THE UTILIZATION SECTION, 12TH FEBRUARY, AND THE MERSEY AND NORTH WALES CENTRE AT CHESTER, 16TH MARCH, 1959

Mr. G. E. Skillington: The data on discrimination under short-circuit conditions are most valuable, since in the past the lack of this information has caused users to employ larger ratios of current rating between major and minor fuses than are apparently necessary, with a resultant waste of money.

Regarding the author's reference to simplicity of construction and consistency of manufacture, I feel that far too much manual work is still associated with the manufacture of fuse-links.

I agree the author's comments on system planning and layout, but consider that he should also have called for discrimination and certainty of fault clearance under earth-leakage conditions.

When working inwards on the design of a 3-phase 4-wire distribution tree protected throughout by h.r.c. fuses and using the author's 1 : 2 ratio between successive minor and major fuses, one can arrive at the need for some very large h.r.c. fuses at the supply end of the tree.

Since the Nullung earthing system is not used in this country, it may prove very difficult to reduce loop impedances to values which will permit a current equal to three times the fuse rating to flow and operate the fuse element in a total time short enough to avoid serious shock and fire risks. If a fuse does clear the earth fault, one is inevitably left with the problem of sustained operation on two phases. The 3-phase tripping switch-fuse with core-balance transformer and spill-current fuse to effect rapid clearance of earth faults and limit earth-fault currents once seemed to be the answer to all these problems, but the inconsistent operation of the explosive charge seemed to destroy users' confidence and such equipments never came into general use. Successful equipment of this kind is available in other countries. Are such units likely to be manufactured and widely used in this country? I am convinced that the need for them is very real throughout industry.

To ensure consistency in manufacture, does the author consider it an advantage to check fuse resistance both cold and at rated current? Moreover, to prove non-deterioration, are not tests at, say, 80% load for say 1 000 hours advisable?

The author states that sufficient margin must exist between the total operating time of the minor fuse and the pre-arcing time of the major fuse to ensure that the major fuse element is not permanently affected by the I^2t which has to pass through it to blow the minor fuse. No indication is given in the paper of the necessary margin, and the author's views would be appreciated.

The author refers to a critical value of prospective current and says it is not necessarily the highest. More information on this point would be helpful.

The author deprecates the usefulness of the virtual-time method of showing fuse performance. I do not share his views and would be interested to know the views of other fuse designers in this connection.

The author states that time/current characteristics are lines representing the mean values of tolerance bands, but he gives no indication of the width of the bands. Can he give these for different fuse ratings?

Can the author produce derating data for main fuses continuously loaded in ambient temperatures higher than 25° C?

Mr. T. B. Rolls: Since even B.S. 88 is not enthusiastic about the abbreviation h.b.c., the author is right in adhering to h.r.c. until something better can be agreed.

I regard the paper as a challenge to see how far cartridge fuses can be used. Fig. A(i) shows how cartridge fuses on the l.v. side of a transformer discriminate with an overcurrent relay of the inverse-time type having a characteristic in accordance with B.S. 142. Discrimination on heavy short-circuits is no problem, but lesser faults at some distance from the substation limit the maximum size of fuse to about 300 amp for this example.

If a fuse were to be used on the h.v. side instead of a circuit-breaker, Fig. A(ii) shows that the largest fuse which could be used on the l.v. side of the transformer would be 600 amp. New designs of relay are, however, available having 'very' or 'extremely' inverse characteristics whose curves are better suited for discriminating with the h.r.c. fuses on the l.v. side of the transformer, so avoiding the size of the latter being unduly restricted by the needs of discrimination. Time-lag fuses provide similar, but less reliable, characteristics.

Mr. J. R. Anderson: The author stresses the need for consistency of manufacture and non-deterioration. It is clear that for close discrimination the curves must be consulted and that these must be available for the manufacturer's range of fuses. However, since usually the 2 : 1 ratio will be satisfactory, it might be advantageous to adopt some simple system of notation for fuses, to indicate to a works electrician which fuse would discriminate with another.

The author mentions that on test a major fuse had survived several through short-circuits. Would it not be more likely for deterioration to occur with a series of sustained overloads near the fusing factor—a condition which might well occur in practice under emergency conditions? Would preheating under such conditions have any significant effect on the discrimination?

An example of the badly planned lighting circuit which the author has in mind in Section 5.4 would be of interest.

I would endorse his remarks in connection with discrimination on distribution systems, since in a large urban system using 500 or 1 000 kVA transformers, discrimination is achieved more or less automatically, the distributor fuse being 400 or 500 amp; and since the largest load supplied from an m.v. system will be approximately 100 kVA, the fuses in the cut-out on the consumer's premises would not exceed 200 amp. With small domestic consumers using 60 amp fuses in the cut-out, discrimination is good, even if the consumer's fuses are of the 30 amp semi-enclosed type. Difficulties arise with cartridge fuses

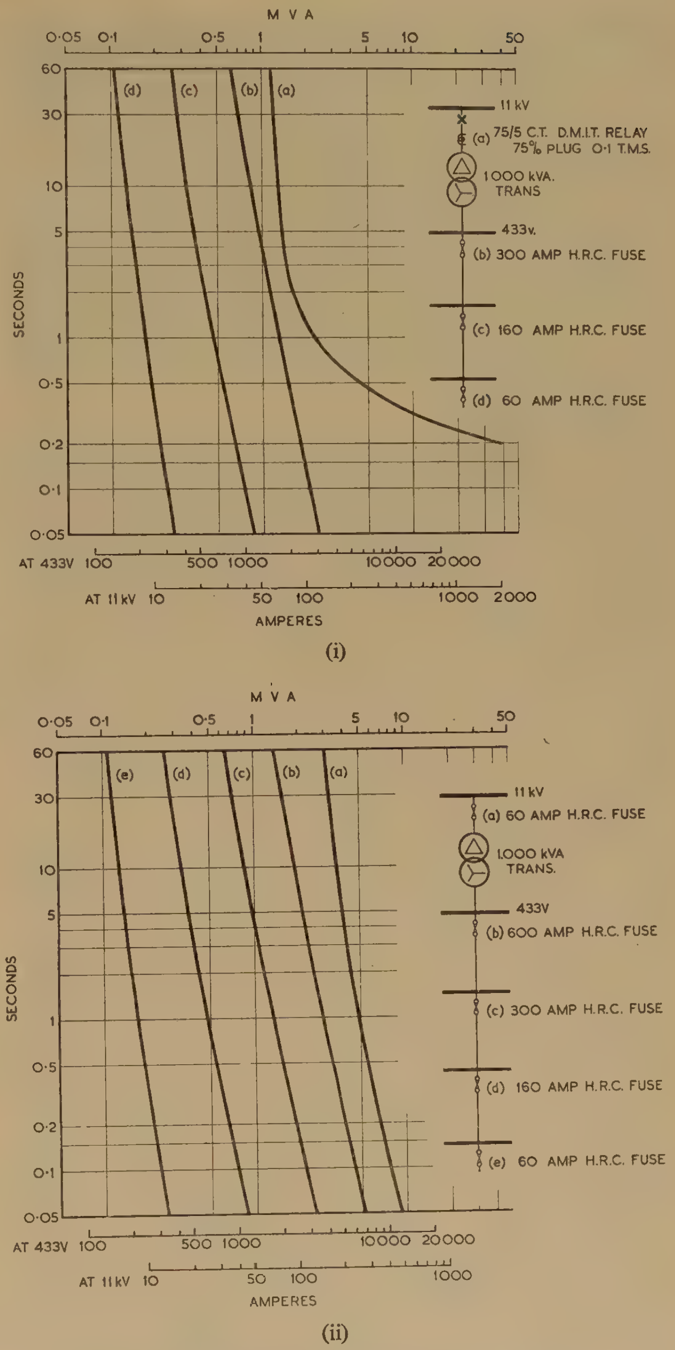

Fig. A.—Discrimination for 3-phase short-circuits.
(i) Cartridge fuses on l.v. side.
(ii) Fuse on h.v. side.

in consumer units, where there is a tendency to indulge in make-shift replacements.

An extension in the scope of the paper to include rewirable semi-enclosed fuses would have been of considerable interest and value.

Mr. R. H. Dean: The author states that the principal duty of the fuse is to interrupt short-circuits and that discrimination is usually regarded as a secondary requirement. I should like to challenge this. I think the duty of a fuse is to protect the apparatus it is serving on all the faults which may occur, i.e. from 25% overload after a long period of time up to the heavy

faults for which high breaking capacity is required. The time for it to operate on any particular overload should follow the time which the hot spot in the apparatus it is serving takes to rise to damage temperature and should discriminate with it. The Institution's Wiring Regulations recognize this to a certain extent in Rule 316, which states that the current rating of the fuse must not exceed that of the cable.

No description of the particular fuse discussed is given. This is unfortunate, since the paper gives the incorrect impression that all h.r.c. fuses have similar characteristics. There are two main groups—the dual-element type, consisting of a time-lag zone in series with the h.r.c. portion, and the silver-wire type which the author is discussing. These fuses have very different characteristics, particularly in the overload zone, and the author might comment on the problem of discrimination between the two groups of fuses.

Tests demonstrating non-deterioration are widely mentioned in the paper, but no particulars are given. For such an important point—and the whole validity of the paper rests on the fuses remaining consistent—I regret that the author gave further details only in slides on the screen. However, I agree that the criteria of success should be the cold resistance of the fuse before and after the tests. The tests shown, however (apparently very successful), covered only fault current down to 8 kA. The usual causes of deterioration are not high fault currents but continuous overloads and surges such as motor starting direct on line, and in this region the tests are incomplete. This is quite an important point, since silver-wire fuses to class Q have so far been thought to be non-deteriorating only up to the full-load rating. It is not generally realized in the industry that non-deterioration is claimed on continuous overloads or surges.

Mr. W. J. Elliott: I^2t, in some form, is a necessary device for comparing fuse performance at high fault levels. With currents which produce fusion in very short times, considerable variation in cut-off current and in actual time can be obtained for a given prospective current by variations in asymmetry due to differences in the points on the voltage wave at which the circuit is closed. This results in considerable scatter of points of time/current relationships at even longer times than the 0·02 sec at which the curves in Fig. 6 start. Some conventional means of plotting the early parts of these curves must have been used, and I suggest that this was virtual times. Would it not be better to continue the curves to lower times by the use of virtual times, obtained by dividing I^2t by the square of prospective current, and so provide a continuous record rather than end the curves at 0·02 sec and quote I^2t separately? There are indications that I^2t is not a constant, and that even at very short times much of the energy supplied is lost by radiation and conduction. The curves in Fig. 8 show an increase in I^2t, from 0·01 to 0·05 sec, of over 100%. This variation can be taken into account with virtual-time curves, but not by the author's method of expressing a fixed value for I^2t. Curves of total operating time and pre-arcing time can be plotted on separate sheets, thus avoiding the difficulties of presentation mentioned by the author.

In connection with the effect of unequal loading on discrimination, as illustrated in Fig. 9, the author states that this is a minor factor when the fuse elements are designed to run relatively cool. Would he agree therefore that, from this point of view, more consistent performance would be expected from class-Q than from class-P fuses?

Mr. H. Simmonds: Having been closely associated with the manufacture and utilization of h.r.c. fuses for many years, it is my opinion that, although the high-rupturing-capacity characteristic of these fuses is an essential one, it is their property of discrimination more than any other that has brought them into widespread use. The vital importance of this property of discrimination is that, when h.r.c. fuses isolate a faulty circuit, healthy circuits fed from the same supply source retain their supply without interruption.

The author is correct in using the term 'h.r.c.'. This term is in almost universal use to denote the type of fuse concerned. I consider it most unfortunate that the term 'h.b.c.' was used in a recent paper on fuses.* Why this attempt to change the established term? It has caused confusion, particularly oversea, and has given the impression that it had been decided to change the term 'h.r.c.', whereas nothing could be further from the truth. With the issue of a separate British Standard for semi-enclosed fuses, an opportunity will now present itself to incorporate the general term 'h.r.c.' in B.S. 88 and so regularize what has already been established by common usage. A precedent for this exists in the Canadian Specification No. 106 which is entitled 'Construction and Test of H.R.C. Fuses (Low-Voltage Power Fuses)'.

Finally, I would emphasize the author's comments on the importance of non-deterioration in h.r.c. fuses and the necessity for high standards of manufacture.

Mr. J. W. Gibson: The author has exploded the fallacy that for discrimination the total operating time of the minor fuse must be less than the pre-arcing time of the major fuse; Fig. 2 illustrates that this is in no way necessary. Mr. Rolls shows that the problem of discrimination between the protective means on the two sides of a step-down transformer is simplified if fuses are used on the primary side also. Discrimination can then be predicted by the author's method, modified simply by taking into account the turns ratio. If the transformer is delta/star connected, a secondary line-to-line fault gives rise to a current through one primary fuse 15% greater than determined simply from the ratio, but the effect of this on discrimination is outweighed by the reduction in arcing I^2t on the secondary side resulting from the presence there of two fuses in series.

I prefer virtual-time curves for predicting discrimination, both for convenience and because they take account of the fact that, even at short times, I^2t values for both the pre-arcing and the arcing periods are not by any means constant for a particular fuse. I cannot agree with the statement in Section 3.5 that the arcing I^2t is likely to reach a maximum at an intermediate prospective current. If it did, and assuming that Fig. 7 is drawn for a particular prospective current, e.g. 46 kA as for Category AC5 of B.S. 88, the two lines in this Figure would not converge to the right. The practical significance is that the greatest fault current requires the largest margin for discrimination.

It would be useful if the author would give quantitative recommendations regarding:

(*a*) The margin (see Section 3.3) between I^2t values to avoid damage to the major fuse.
(*b*) The width of the tolerance band (see Section 3.13) on time/current characteristics, bearing in mind that B.S. 2692 gives ±20% for h.v. fuses.
(*c*) The effect on discrimination (see Section 4.1) of the major fuse being hot; information might be given as a curve relating pre-arcing I^2t to pre-loading current expressed as a percentage of rated current.

Mr. H. W. Baxter: The author plots the characteristics in Figs. 6 and 8 in terms of prospective current. This can be misleading, because it is not the same as the effective current which flows through the fuse and which is important from the standpoint of discrimination.

Prospective current relates to the circuit (it is the current which would flow if the fuse were replaced by a link of negligible impedance), and is used in testing a device for breaking capacity;

* DEAN, R. H.: 'Recent Developments in Medium-Voltage H.B.C. Fuse Links', *Proceedings I.E.E.*, Paper No. 2419 U, October, 1957 (105 A, p. 263).

Fig. 8 shows values of prospective current up to 100 kA, corresponding to values of circuit impedance down to about $2\frac{1}{2}$ milliohms. A symmetrical current of 100 kA r.m.s. would have an initial rate of rise of about 44 MA/sec. A small fuse can have an impedance of several milliohms, and the insertion of this impedance would reduce the rate of rise of current to a value lower than the prospective current of the circuit would indicate. The fuse would therefore take longer to blow. Consideration of an extreme case of a small radio fuse of 1-ohm impedance shows that this would limit the current to 250 amp in a 250-volt circuit, even if the prospective current of the circuit were 100 kA. I consider that prospective current is not the right quantity to use in this case, because the fuse is melted by the actual current and not by the current which would flow if the fuse had no impedance. The impedance of the fuse cannot always be neglected in determining the current which will flow through it. Characteristic curves plotted on a basis of prospective current cannot strictly be used to determine whether one fuse will discriminate with another if the actual current is not the same in each case. It might be appropriate to introduce a term, 'virtual current', since this is compatible with virtual time and is readily calculable from an oscillogram.

Mr. K. Dannenberg (*communicated*): I should like to draw attention to two omissions by the author which have considerable bearing on the question of discrimination. B.S. 88 prescribes the pre-arcing time to be plotted from the no-load state. In practice, such no-load conditions would hardly ever apply, so that knowledge is required by the user relative to the variation of the no-load pre-arcing time/current characteristics with various loadings up to full load. With some well-known fuse makes the variation of pre-arcing time when the fuse is subjected to fault currents between 2 and 4–5 times its continuous current rating averages up to 300%. A 300 amp fuse would melt when subjected to a fault current of 900 amp in approximately 5·5 sec when loaded 100% prior to the over-current, but would take 18 sec to melt when subjected to the over-current from no-load conditions. The obvious conclusion must be that each line of the time/current curves becomes a wide band within the range of multiples up to five times the current rating of the fuse, which in service may be the most critical range from a discrimination aspect. Once it is clear that this wide band applies to any one design, the ratio of 2 : 1 suggested by the author in Section 2.2 becomes for practical reasons a minimum.

A great deal has been published about the difficulty of discriminating between fuse links of different manufacture—although similar in design—when installed in the same fuse-holder. Bearing in mind that for each link, for the reasons referred to above, the discrimination range could not be less than 2 : 1 in practice and is in fact determined by a very wide band in the over-current range, this problem is considerably simplified.

The author has omitted to mention the thermal tripping device which introduces a measure of overload performance to the h.r.c. fuse link, quite distinct from obtaining this by a change of fusing factor. Most extensive improvements have taken place in the application of the thermal trip by employing materials which ensure solid electric joints and a circuit where accurate predetermination of tripping operation can be effected. The combination of the h.r.c. fuse with an isolator actuated by such a thermal trip represents a very important achievement extending considerably the range of protection as achieved by discriminatory action of h.r.c. fuse links.

Mr. G. L. Simpson (*at Chester*): There is still considerable lack of appreciation of the purpose of h.r.c. fuses, and the author rightly refers to the necessity for a properly planned system; but this cannot be achieved so long as users continue to regard h.r.c. fuses as overload devices rather than short-circuit devices. The manufacturers' practice of publishing time/current characteristics in the present form does not help, and it would appear that the most useful criterion upon which fuses should be selected is the I^2t method referred to in the paper.

It is interesting to note from Fig. 7 that the 300 amp fuse and probably the 500 amp one are not of much value for discrimination, and if the former were dropped and a 600 amp fuse substituted for the latter, a more consistent family would result.

There would appear to be a strong case for some rationalization within the industry with a view to standardizing fuse characteristics and codes of practice, with perhaps some form of numbering or colour coding for discrimination, although even this might be unnecessary with properly selected standard ratings so arranged that every fuse discriminated with those of higher value.

Mr. E. Jacks (*in reply*): Both Mr. Skillington and Mr. Gibson confirm the necessity for consistency in manufacture. This is a matter for each fuse manufacturer to decide in respect of his own fuses; the important thing is that the width of the tolerance band should be known to the user, and experience has shown that it is practicable to manufacture h.r.c. fuses to $\pm 5\%$ except for the very small current ratings.

Regarding Mr. Skillington's remarks on earth-leakage protection, I contend that h.r.c. fuses in combination with suitable earthing arrangements have been proved to be the best general proposition for earth-fault protection (as distinct from earth-leakage protection) throughout industry.

Information concerning critical values of prospective current may be found in E.R.A. Report G/T302. The derating of fuses for continuous loading in ambient temperatures higher than 25° C depends largely on the method of rating, which in B.S. 88 is based upon arbitrary tests. The real problem is to relate such tests to service conditions, and this does not permit of a simple and unqualified answer to the derating question.

Mr. Rolls's contribution is a valuable addition to the paper. I should, however, like to emphasize his own words that it is only at lesser faults at some distance from the substation that the maximum size of fuse needs to be limited to 300 amp. Fuses in close proximity to the substation busbars are normally subject only to the higher short-circuit faults and can have current ratings considerably higher than 300 amp.

Messrs. Anderson and Dean are concerned about the effect of sustained overloads in the region of minimum fusing current, and in his recent paper, Mr. Dean recommended tests including one at 95% of minimum fusing current for 40 hours. Class-Q fuses, having reasonably homogeneous elements, have been tested at this value for several hundred hours, as well as withstanding the other tests without showing evidence of deterioration.

The sort of badly planned lighting circuit which I had in mind when writing Section 5.4 was that in which the lighting circuits are mixed with power circuits and tapped off in a similar manner to that shown in Fig. 10 (circuits E and F).

I cannot agree with Mr. Dean that all fuses should necessarily provide overload protection where no electrical fault exists. This is normally done by other means, particularly in motor circuits. Nevertheless, Class Q-fuses properly rated and applied will give overload protection to cables when these are installed in accordance with The Institution's Wiring Regulations.

I have refrained from giving a description of the particular fuse with which I am most familiar because this does not directly affect the principles of discrimination, which apply for any type of fuse provided that the necessary data are available.

I do not disagree with Mr. Elliott's remarks concerning the presentation of I^2t data. The question is really one of practicability from the user's point of view. The I^2t values shown in

Fig. 7 of the paper are limiting values which are useful for the evaluation of short-circuit stresses as well as for discrimination. I feel that the information given by Mr. Elliott is likely to be useful more to the specialist than the average user.

I regret that I have no data readily available concerning the effect of unequal loading on class-P fuses. My remarks in the paper are based on experience of class-Q fuses.

Mr. Gibson is not correct in assuming that the total operating I^2t values shown in Fig. 7 are all drawn for a particular prospective current. As stated in Section 3.6, these are maximum values which occur at the critical prospective current in respect of each fuse rating.

The effect on discrimination of the major fuse being warm due to preloading is not very important when considered against the margin referred to in the final sentence of Section 3.6. Extensive tests have shown that proper discrimination can be obtained at the lower fault values between cold minor fuses having ratings up to 80% of the warm major fuses. Tests have been carried out in which the major fuse has been preloaded up to 100% for several hours, after which a cold minor fuse (at an ambient temperature) has been switched in the circuit on to an applied overload of 300%. The tests have been repeated several times using the same major fuse in successive shots. The criterion of success was that no deterioration of any kind could be detected in the major fuses after the tests were completed. These tests are more severe than any condition which could arise in service, because it is not readily conceivable that additional load would be applied through a cold minor fuse on to a major fuse which was already carrying 100% load.

Mr. Dannenburg quotes the case of certain fuses whose time/current characteristics are considerably affected for the longer operating times by preloading. This is true of some fuses whose characteristics become asymptotic at the longer times, but even with these the effect of preloading on discrimination is not serious. It must be remembered that the power loss for a given current will be greater in the minor fuse because of its higher resistance, and this, coupled with the fact that a working margin is almost always allowed in practice, usually results in successful discrimination with ratios much less than 2 : 1

I appreciate Mr. Baxter's plea for the correct usage of the term 'prospective current' but doubt whether another term is necessary except perhaps for the fuse specialist.

I am in some doubt as to Mr. Simpson's point concerning the 300 and 500 amp fuses shown in Fig. 7. It must be appreciated that the ratings shown correspond to preferred 'sizes'. Intermediate ratings are, of course, available.

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The Institution of Electrical Engineers
Paper No. 3044 S
Aug. 1959
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ARC MOVEMENT IN A TRANSVERSE MAGNETIC FIELD AT ATMOSPHERIC PRESSURE

By P. E. SECKER, B.Sc.(Eng.), Graduate, and A. E. GUILE, Ph.D., B.Sc.(Eng.), Associate Member.

(*The paper was first received* 18*th June, and in revised form* 8*th October*, 1958.)

SUMMARY

The velocities of d.c. arcs moving in the forward direction in a transverse magnetic field, in air at atmospheric pressure, have been measured at arc currents up to 700 amp, and magnetic fields up to $0 \cdot 1\ \mathrm{Wb/m^2}$.

The results confirm the findings of an earlier investigation[1] where the driving field was due to current flowing in the electrodes, that the arc movement is chiefly controlled by cathode surface processes. It is affected by a variety of factors, some of which have not been sufficiently appreciated or controlled in previous investigations.

The cathode movement and the resulting track on the surface is either continuous or discontinuous. In the case of the former mode, cathode surface phenomena alone are decisive. In the latter mode, even though jumps of random length occur, the movement is still definite and consistent and depends upon a combination of cathode-root and arc-column processes. Thus, the overall arc movement is at all times controlled and repeatable within surprisingly small limits, any scatter being due mainly to differences in cathode surface layers.

Above about 40 amp, the velocity of the cathode root, and thus that of the whole arc, is independent of arc current, whether its movement is continuous or discontinuous.

(1) INTRODUCTION

The movement of an arc in a magnetic field involves complex electromagnetic and thermal forces, which are operative in the arc column, in the cathode and anode roots, and in the electrodes. Although the phenomenon of arc movement by magnetic fields has been known for 130 years and has been the subject of considerable investigation, little is understood of the relative roles of these forces in the various regions, so that very little reliable quantitative data are available to enable arc behaviour to be predicted under any particular conditions. Application of the usual theory of electromagnetic deflection to the simplified arc model, in which the arc is considered as a flexible physical conductor, indicates the correct direction of motion at atmospheric pressure but is unable to predict the values of velocity observed, and can in no way account for the fact that, as the pressure of the atmosphere surrounding the electrodes is reduced, the arc slows down, stops, and may move in the reverse direction.[2, 3] This retrograde motion may even occur at low currents at atmospheric pressure.[4]

Some previous investigations,[1, 5] where the driving field was due to current flow in the electrodes and to magnetization of the electrodes, have shown that the forward motion of the arc when it leaves a continuous cathode track is largely controlled by conditions in the cathode-surface or cathode-fall regions. Several theories, proposed to explain retrograde motion, have also relied on processes occurring, not in the column, but in the cathode-fall region. The study of arc movement has now been extended to the case where the arc is driven in the forward direction by an externally applied transverse magnetic field, and the field due to current flow in the electrodes has been eliminated.

There has previously been little discussion of the mechanism of arc movement in the forward direction and little systematic investigation. It will be shown that previous investigators of forward movement in a transverse field[6, 7, 8, 9] have been unaware of some of the factors which affect the movement, so that the conditions of their experiments have not been sufficiently controlled. The results which are given here show that an explanation of forward arc motion cannot be attempted as hitherto, merely by studying the macroscopic conditions in the arc column, but that the arc progression is largely determined by the properties of the microscopic emission sites at the cathode surface.

(2) EXPERIMENTAL PROCEDURE

The electrodes, which were set up horizontally and parallel, one above the other in the same vertical plane, were, for the majority of the tests, cylindrical, of 9·5 mm diameter. The current was fed in at both ends of the anode, which was mounted above the cathode; current from the latter was taken equally from both ends by adjustment of the balancing impedances, as shown in Fig. 1(*a*). This eliminated the transverse self-magnetic field due to current flow in the electrodes, such as occurred when the arrangement in Fig. 1(*b*) was employed.[1] Thus, the arrangement in Fig. 1(*a*) enabled a magnetic field to be applied which was independent of the arc current, provided $Z_1 = Z_2$.

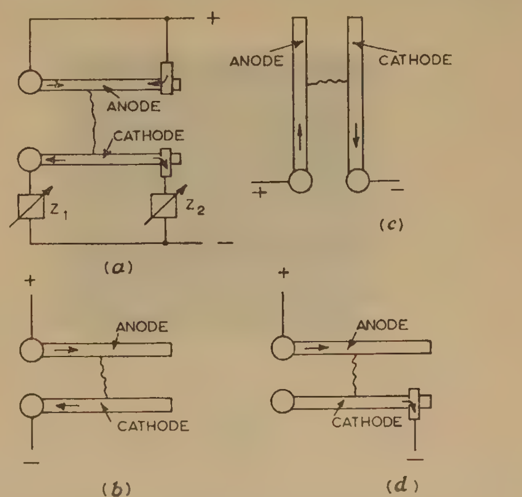

Fig. 1.—Electrode arrangements.

(*a*) Double-end current feed.
(*b*) Single-end current feed.
(*c*) Single-end current feed, with electrodes vertical (Eidinger and Rieder).
(*d*) Current feed to reduce self-fields (Winsor and Lee).

Written contributions on papers published without being read at meetings are invited for consideration with a view to publication.

Mr. Secker is, and Dr. Guile was formerly, in the Department of Electrical Engineering, Queen Mary College, University of London. Dr. Guile is now in the Department of Electrical Engineering, University of Rangoon, Burma.

The electrode supports were mounted so that the gap between the electrodes could be varied. The arc was initiated about $\frac{1}{2}$ in from one end of the electrodes by the explosion of a fuse wire connecting them, and was then driven by the transverse flux density set up by a large solenoid, at the centre of which the electrodes were situated. The flux density at the centre of the solenoid could be adjusted to any value up to the maximum of $0 \cdot 14\,\mathrm{Wb/m^2}$, and over the whole central cross-section was found to have a maximum variation of $\pm 3\frac{1}{2}\%$ from the mean value.

The arc movement was recorded on a drum camera operating at 960 frames/sec, and the current in each cathode lead, the arc voltage and the solenoid current were recorded on a Duddell oscillograph, together with a 50 c/s timing wave.

The arc currents, up to the maximum of 700 amp, were supplied from a 500-volt d.c. machine.

(3) CATHODE ROOT MOVEMENT IN A TRANSVERSE MAGNETIC FIELD

(3.1) Modes of Cathode Movement

As has been found in certain previous investigations,[1,7] several different modes of arc movement may occur. Different types of track are left on the cathode and some arbitrary means of classification must be adopted. Four types of track were observed:

Discontinuous track.—(*a*) The cathode root appeared to have dwelt at a few points only, whilst the rest of the electrode was unmarked.

Continuous track.—(*b*) The continuous track showed evidence of some surface melting—regular track.

(*c*) Very severe melting occurred and the arc root moved very slowly—sticking track.

(*d*) The track was continuous but presented a branched or fernlike appearance and produced so little surface marking that the electrode could sometimes be restored almost to its original condition by rubbing with a soft cloth—high-speed regular track.

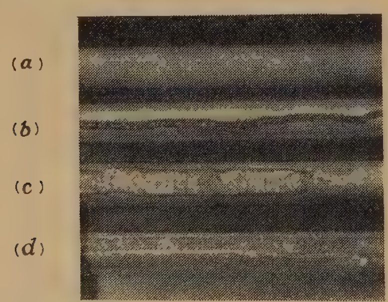

Fig. 2.—Types of cathode arc track.
(*a*) Discontinuous.
(*b*) Regular.
(*c*) Sticking.
(*d*) High speed.

Fig. 2 shows an example of each of the above tracks. Tracks (*b*), (*c*) and (*d*) were seen to have certain common characteristics, it being probable that (*c*) and (*d*) represented the lower and upper velocity limits respectively of (*b*). Thus, two main modes of motion were evident—the discontinuous motion of track (*a*) and the continuous motion of tracks (*b*), (*c*), and (*d*). Results of velocity values in the two principal modes of motion are given in the following Sections together with a discussion of the factors affecting the mode of motion which is operative.

Discontinuous movement, type (*a*), has previously been referred to[1] as random movement. The distances between marks on the cathode are quite random, but it will be shown here that the velocity in this mode is consistent and repeatable.

(3.2) Continuous Movement of the Cathode Root

At $3 \cdot 2$ mm spacing on clean untreated mild-steel electrodes, about 90% of the cathode movement was continuous regular track, i.e. mode (*b*), and the rest was discontinuous. The regular velocities did not have a unique value for a given magnetic flux density, but lay within very clearly defined limits, with a variation of about $\pm 50\%$ of the mean value, as shown in Fig. 3.

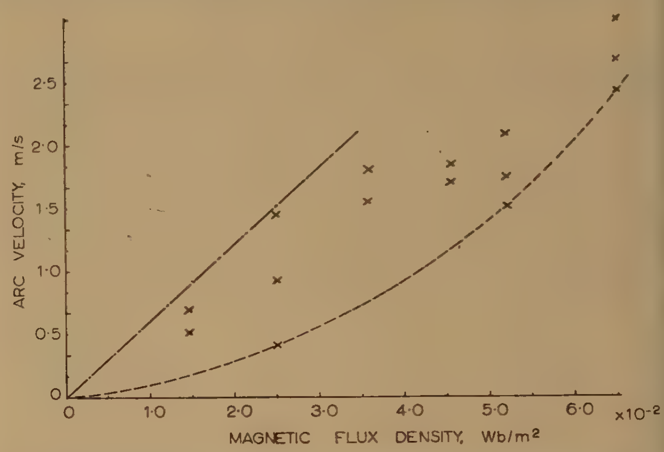

Fig. 3.—Scatter in the regular velocities of the cathode root on mild-steel electrodes $3 \cdot 2$ mm apart at an arc current of 330 amp.

The points plotted are single test results, and not mean values of a number of tests. For a given flux density, there is a maximum value of velocity which is independent of the arc current over the range 40–670 amp, as shown in Fig. 4. In any one test, the whole range of regular velocities sometimes occurred as the roots traversed an electrode, on which slightly different surface conditions could be distinguished. It appears, from results described later, that this variation in values of regular velocity was due to surface layers on the cathode.

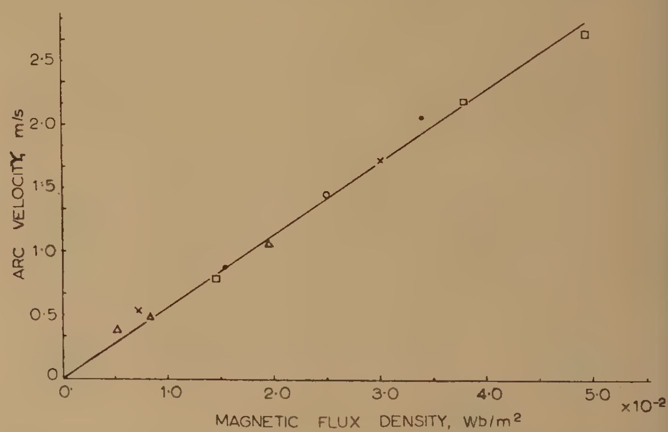

Fig. 4.—Maximum velocity of cathode root motion on mild-steel electrodes, $3 \cdot 2$ mm apart.

Arc currents:
× 43 amp.
△ 64 amp.
● 103 amp.
○ 335 amp.
□ 670 amp.

When only lightly tarnished untreated brass was tested, the

scatter in regular velocity was greater than that for steel, at velocities somewhat above those on the latter. However, when the brass was polished with a proprietary cleaning fluid, the mode of motion changed from regular to high speed and the scatter was reduced from greater than ±50% to within ±5% of the mean value (see Fig. 5). Again no dependence of velocity

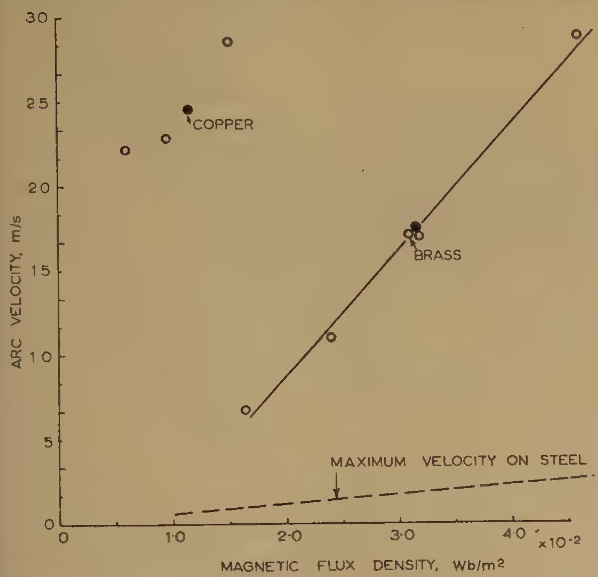

Fig. 5.—Velocities of cathode root on polished brass and copper electrodes, 3·2 mm apart.

Arc currents:
○ 500 amp.
● 170 amp.

on current was observed over the range 40–670 amp. Below 40 amp the velocity was found to decrease as the current was reduced, and the track form changed from the high-speed track to a thin regular track which had a marked tendency to become branched for short sections. Continuous velocities in mode (*b*) or (*d*) could not be obtained on highly tarnished brass electrodes, but sticking tracks (*c*) occurred, giving velocities between 0·075 and 0·3 m/s at fields up to 0·05 Wb/m². Occasionally this mode of motion would give way to discontinuous motion, mode (*a*). When heavily tarnished electrodes were polished with cleaning fluid, the velocity increased to about one hundred times its former value as the track changed from (*c*) to (*d*). Velocities on untreated lightly tarnished brass have been found which in the one case lie on the curve of Fig. 5 for polished metal, and in the other lie on that for the sticking track on the highly tarnished brass.

On copper electrodes all types of track except the sticking track were observed. Regular tracks which occurred below flux densities of 0·005 Wb/m² seemed to have velocities which were not highly reproducible, whether the copper was polished or not. On polished copper above this value of field, values for velocity of high-speed tracks showed far less scatter (see Fig. 5). The limited range of this high-speed velocity curve is due to the fact that arc velocities above about 33 m/s could only have been measured by a camera operating at a speed well above the 960 frames/sec of the camera available.

Tests on both carbon and aluminium electrodes gave hardly any regular tracks, so that results for these materials are not shown. On aluminium, when the flux density was too low to cause random jumping (below about 0·01 Wb/m²), the arc tended to stick completely. In some cases the arc was blown out, the column being extended by the magnetic field until the circuit voltage could no longer maintain the length of column. When the arc was not so extinguished, a large crater was blasted at the point on the cathode where the root stuck.

In contrast to the absence of regular tracks on aluminium in an external field, most of the tracks on aluminium due to the self-field set up by current flow from one end of the electrodes only were found to be regular.[1] Use of this property has been made to investigate the relation between flux density and cathode velocity by exploring the region between current flow from one end only, and equal current flow from opposite ends. Tests were carried out on aluminium electrodes, with the current-limiting impedances Z_1 and Z_2 in Fig. 1(*a*) deliberately unbalanced. No external field was applied, so that the arc moved owing to the resultant field set up by the current difference, I_1-I_2 (see Figs. 1 and 6). The inter-electrode gap was set at 3·2 cm, so that the arc velocities could be compared with those previously established during tests with current connections at one end of the electrode system only.[1] The currents used were above those at which the velocity of motion was found to be dependent on the current. Employing the simple current-flow pictures of Figs. 6(*a*) and (*b*) to calculate the self-fields, it would

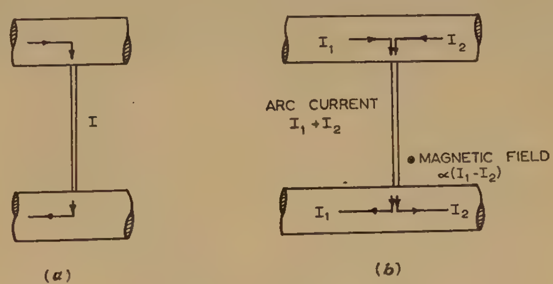

Fig. 6.—Current flow patterns.

(*a*) With single-end feed.
(*b*) With double-end feed.

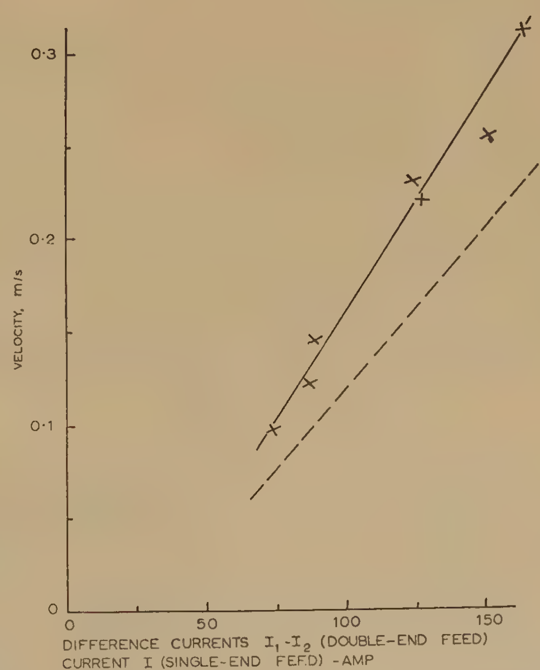

Fig. 7.—Regular cathode velocity on aluminium electrodes, 3·2 cm apart, for single-end and double-end current feeds.

——— Double-end feed.
– – – – Single-end feed.

be expected that the velocities of arc travel would be identical if, for double-end feed, the difference current, I_1–I_2, were made equal to the current, I, flowing from one end only. Fig. 7 clearly shows that this is not the case. Obviously, the current-flow patterns shown in Fig. 6 will be distorted near the arc root. This distortion may well lead to widely differing magnetic flux densities at the cathode surface for the two cases, whereas calculations based on the flow patterns of Fig. 6 give identical values of flux density. In the arc column some distance from the cathode surface it is clear that the current-flow distortion in the electrodes will have little effect on the flux density. Thus, the difference shown in Fig. 7 supports the hypothesis that the seat of arc movement is at or near the cathode.

(3.3) Discontinuous Movement of the Cathode Root

The mode of arc movement in which the cathode root left small marks scattered at irregular intervals gave velocities which, when plotted against flux density, yielded a continuous curve, and it is surprising that the scatter was somewhat less than that for regular velocity, being about ±30% for steel. These velocities were obtained by measuring the time taken for the cathode root to move by a number of jumps over a total distance of some 5–13 cm. It was found convenient to plot the velocity and flux density on logarithmic scales. Fig. 8 shows the curves

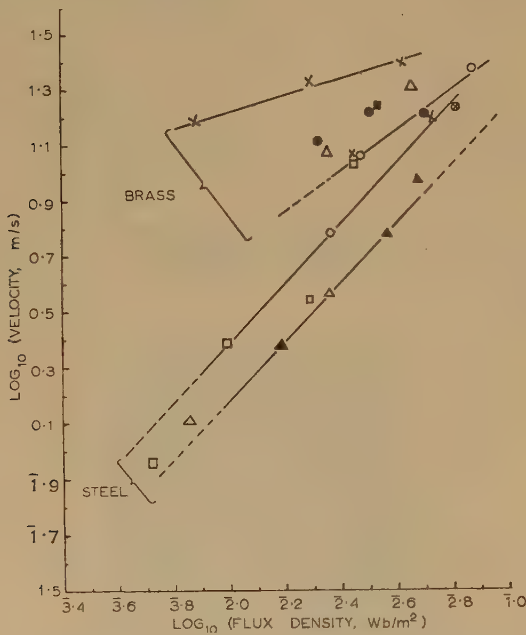

Fig. 8.—Limiting curves of discontinuous cathode velocity on steel and brass electrodes, 3·2 mm apart.

× 30 amp. △ 40 amp. □ 65 amp. ▲ 100 amp.
● 170 amp. ○ 235 amp. ■ 300 amp. ⊗ 660 amp.

for brass and steel. While all the steel electrodes were in a clean untreated state, the brass electrodes varied between the extremes of heavily tarnished rods to those which had been polished with the proprietary cleaning fluid. This probably explains the wide scatter in the results for brass.

It was found that, when discontinuous movement occurred on polished brass electrodes, the velocity lay near the upper limit given in Fig. 8 for any particular flux density. On lightly tarnished electrodes the velocity lay near the lower limit, whereas on heavily tarnished brass the velocity could assume any value between the two limits.

It will be observed that, just as for regular velocity, the velocity does not vary with the arc current over the range 40–670 amps while below 40 amp, random tracks have not been found to occur. In Fig. 9, the maximum random velocity curves have

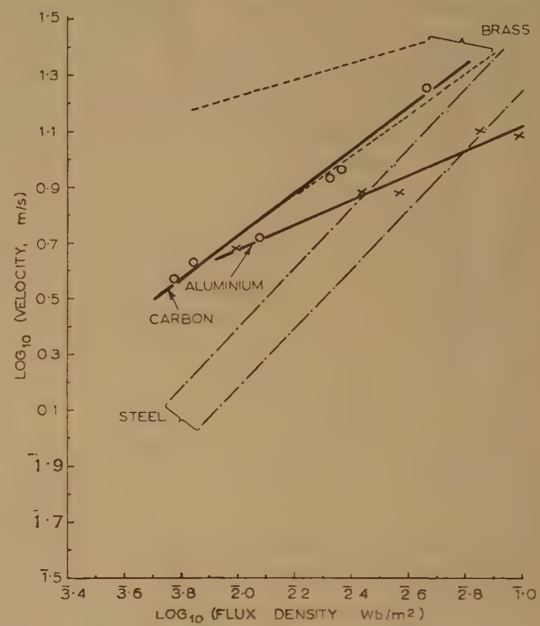

Fig. 9.—Discontinuous cathode velocities on carbon and aluminium electrodes, 3·2 mm apart.

○ Carbon.
× Aluminium.

been plotted for brass and steel, together with the curves for carbon and aluminium. With the latter two materials, the total scatter from the mean curves is less than ±5%. The arc velocity may be written in the form:

$$V = kB^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where k and n are constants which are dependent on the cathode material and surface. Table 1 gives the values of the constants,

Table 1

Constants of the Velocity Equation

Material	k		n	
Aluminium	34		0·43	
Carbon	155		0·74	
	Maximum velocity	Minimum velocity	Maximum velocity	Minimum velocity
	m/s	m/s	m/s	m/s
Brass	59	137	0·28	0·72
Steel	325	209	1·05	1·05

which, when substituted in the above equation, give the velocities in metres per second for B in webers per square metre.

Tests were also carried out on copper, but the results were very widely scattered so that no useful information could be obtained. It appears to be characteristic of copper for the results to show considerable scatter, both for discontinuous and continuous movement.[1] In view of the evidence given above of the effect of surface conditions, it is possible that a close investiga-

tion of the surface layers on the copper cathode may lead to an explanation of this behaviour.

(3.4) Factors affecting the Mode of Cathode Motion

It has been found that the mode of cathode movement is influenced by several factors:

(*a*) Current.
(*b*) Transverse flux density.
(*c*) Cathode material and surface condition.
(*d*) Inter-electrode gap.
(*e*) Nature of the circuit.

(*a*) In general, it was found that, as the current was increased, the arc passed through the modes sticking track—regular track or high-speed track—random track. It is, however, possible for other factors to reverse this order. On lightly tarnished brass electrodes, increasing the current above that for regular movement tended to cause the cathode root to 'dig in', and this aided the formation of a sticking track, whereas on polished brass, increase of current seemed to favour the establishment of the high-speed track. Thus, variation of current can cause a change from one mode to another with a consequent sudden transition from one velocity to another, but change of current whilst in any one mode does not give a continuous variation in velocity, except below 40 amp.

(*b*) The effect of the transverse flux density at the cathode was very marked. As the flux density was increased, the order of the modes was the same as generally occurred when current was increased, namely sticking track—regular track or high-speed track—random track. This mode order was never found to be reversed, although on highly tarnished brass no regular or high-speed tracks were found, the motion changing directly from the sticking track to the discontinuous track. At flux densities below $0 \cdot 015\,Wb/m^2$, the sticking track was found on brass electrodes whatever the surface condition, but between flux densities of $0 \cdot 015$ and $0 \cdot 055\,Wb/m^2$ the high-speed track was formed on polished electrodes, while on the tarnished electrodes either sticking or discontinuous motion took place. At flux densities above $0 \cdot 055\,Wb/m^2$, discontinuous tracks occurred for all surface conditions.

On steel electrodes, no sticking was observed except when the surface was polished with a cleaning fluid. Regular tracks were left on the electrodes for flux densities up to $0 \cdot 055\,Wb/m^2$, above which random movement occurred. In two tests above $0 \cdot 035\,Wb/m^2$ a form of high-speed track was observed, but attempts to repeat the tests only resulted in regular tracks forming.

(*c*) It has already been mentioned that when lightly tarnished brass electrodes were polished, the mode of motion changed from regular to high speed, giving, for the latter, a velocity which was sometimes as much as 100 times that for the former. Highly tarnished brass electrodes after polishing also gave the high-speed track as before, in the flux density range $0 \cdot 015$–$0 \cdot 055\,Wb/m^2$, whereas on highly tarnished untreated electrodes the sticking track or discontinuous motion occurred in this flux density range.

(*d*) Variation of the inter-electrode distance had a marked effect on the mode of arc velocity. As the distance was increased, the percentage of regular movement became less, until finally only random jumping occurred. Fig. 10 shows that this process was very well defined. There was comparatively little continuous movement at gaps greater than $6 \cdot 4$ mm, but some evidence was found for the regular velocity at a $6 \cdot 4$ mm gap being greater than that at $3 \cdot 2$ mm. Definite information on this would require a camera capable of speeds considerably greater than 1 000 frames/sec.

As the gap increased, the velocity in the discontinuous mode

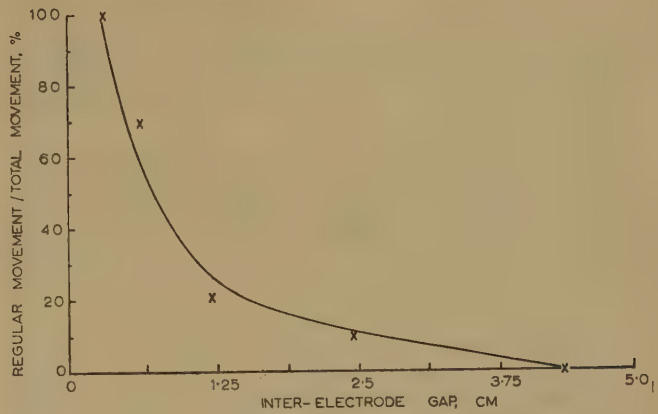

Fig. 10.—Variation of regular movement with inter-electrode gap.

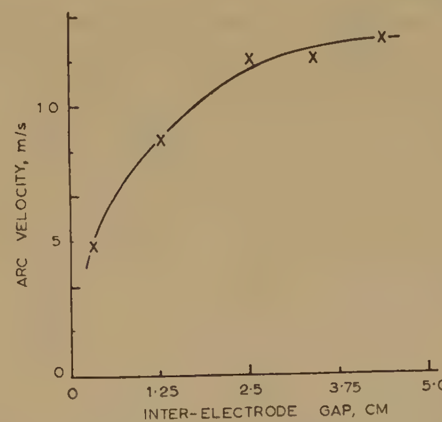

Fig. 11.—Discontinuous velocity on aluminium electrodes as a function of the inter-electrode gap.

increased, but, as shown in Fig. 11, reached a maximum at an electrode spacing of about 5 cm.

(*e*) The current-limiting impedances in the cathode leads could either be made inductive or resistive. When the impedances were made inductive, the arc showed more tendency to move by random jumps, presumably owing to the fact that the arc column could extend more easily than when the circuit impedance was predominantly resistive. It was also found that when the L/R ratio of the circuit was made large, namely $21 \cdot 5$ millisec, the overall arc velocity in the discontinuous mode increased above its value when the ratio was small, namely 2 millisec. An increase in velocity of up to five times was recorded, and for intermediate values of L/R the discontinuous velocities lay between the two extreme values.

(4) PREVIOUS INVESTIGATIONS

The experimental results described above have confirmed that the movement of an arc is controlled by the conditions at the cathode. The velocity of the whole arc may be taken as that of the cathode root, whether the latter is in continuous or discontinuous motion. Confirmation that the anode did not exert a controlling influence was obtained when the cathode velocity was found to be independent of the material of the anode.

Many factors, such as (i) cathode material, (ii) cathode surface condition, (iii) inter-electrode gap, (iv) magnetic flux density, (v) nature of external circuit, and (vi) arc current, have been shown to have some effect in determining the arc velocity, and

it is possible that other factors, such as size of electrodes, nature of gas surrounding them, humidity, etc., may also produce variations in arc velocity. It is well known, of course, that pressure has a pronounced effect on the velocity, since the cathode root frequently reverses its direction of movement at reduced pressures.

In previous investigations, factors (i), (ii) and (v) have not been appreciated and controlled, and (iv) was not always measured accurately owing to an additional field caused by current flowing in the electrodes. Cathode velocities in the continuous and discontinuous modes have not previously been distinguished and the independence of arc current was not found, because lower-speed cameras were used and many tests were made on each electrode. The latter condition prevented a close control of surface conditions and made it impossible to correlate each part of the track on the cathode with the relevant film frames, as has been done here. For these reasons, a close comparison of previous results with those given above is not possible.

Babakov[6] described experiments on plate copper electrodes at gaps between 0·1 and 3 mm, for arc currents of 100–400 amp, and flux densities between 0·01 and 0·1 Wb/m^2. In order to plot continuous curves he found it necessary to take the mean value of six tests because of great variations in velocity. Single-end current connections were used, but it was claimed that the effect of current flow in the electrodes was taken into account. However, it seems probable that the flux density due to the current flow was calculated for the centre of the inter-electrode gap, or that the value used was a mean value over the whole gap, as Babakov considered the arc to move under forces similar to those acting on a physical rod conductor. If this is so, the flux density at the cathode must have been considerably greater than the value he assumed to be operative in driving the arc. The dependence of arc velocity on arc current which he noted could well be accounted for by this fact. Unfortunately, insufficient data on electrode dimensions are given to allow recalculation of the appropriate flux density. A velocity dependence on electrode width was also shown. Babakov suggests that this was due to variation of the friction of the moving gas column on the electrodes. In view of the results now available, it seems more likely that this velocity variation of up to 30%, as the electrode width was changed by 40 : 1, was due to an alteration in the current flow pattern in the electrodes setting up a different flux density at the cathode.

Winsor and Lee[7] carried out tests on silver, copper, aluminium, carbon, tungsten, molybdenum, nickel and titanium electrodes. For regular movement they found no dependence on cathode material. This result is surprising, especially as one of the materials tested, nickel, was ferromagnetic. Our tests have shown, however, that for single-end current feed, a ferromagnetic material has a higher velocity than a non-magnetic one, but when self-field is eliminated by double-end feed, the position is reversed. Since Winsor and Lee had a current feed intermediate between these extremes (current entered the anode at one end and left the cathode at the opposite end), the difference would tend to be masked. It seems likely that the photographic technique used by Winsor and Lee was insufficiently sensitive to allow distinction between discontinuous and continuous movements, since successive exposures only showed the position of the arc at approximately ½ in intervals. Tests with single-end current feed on brass, copper, magnetic stainless steel, tungsten, molybdenum, lead, phosphor bronze, aluminium and mild steel have in fact shown great differences in regular cathode velocities, with no two metals giving the same velocity.[10]

As mentioned above, the electrode assembly set up by Winsor and Lee had the anode lead at one end and the cathode lead at the other, presumably to eliminate the magnetic field due to current flow in the electrodes. However, at the maximum current and minimum flux density values tested, the error in the field at the cathode due to this assumption is shown in Appendix 8.1, by an approximate calculation, to be of the order of 50%. If the approximate correction given by eqn. (3), Appendix 8.1, is used, it is seen from Fig. 12 that no very great difference exists in the

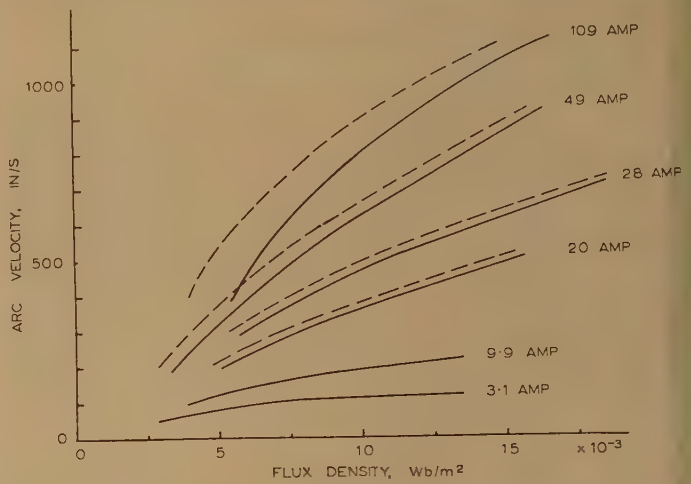

Fig. 12.—Winsor and Lee's results of arc velocity versus flux density on copper electrodes 3·2 mm apart.
---- Quoted value.
—— Corrected value.

curves of velocity against flux density plotted for 109 and 49 amp,[7] whereas below 49 amp the change of current seems to have a marked effect on the continuous arc velocity. This is in agreement with the results of tests which we have carried out at arc currents below 40 amp, which showed that continuous velocity falls with reduction of current. It should be noted that the field correction given in Appendix 8.1 is only approximate, since the evidence of Fig. 7 shows that the non-uniform current flow in the metal near the cathode root should be taken into account. The actual flow pattern of current approaching the cathode spot will produce a field there greater than that indicated by the calculation in Appendix 8.1 based on the current flow shown in Fig. 14. Thus, a rigorous calculation of the field at the cathode spot would tend to bring the curves for 109 and 49 amp in Fig. 12 closer together.

Winsor and Lee found that, unless the surface was oxidized, it was necessary to run an arc along the electrodes several times before consistent results could be obtained. Thus, no surface condition common to all tests seems likely to have been established. When random jumps occurred at currents and flux densities above a certain critical limit, dependent on the material of the cathode, the overall velocity was lower than in the regular-velocity mode. This behaviour has been observed by us on extremely heavily oxidized brass, but in general we have found the discontinuous velocity higher than the continuous velocity, as may be seen by comparing Figs. 4, 5, and 9.

Eidinger and Rieder[8] investigated arc velocity on electrodes, mainly of copper, which were mounted vertically in a manner similar to Fig. 1(*c*), so that the arc was influenced by thermal as well as magnetic forces. Current feed was at the bottom of the electrodes, but it seems probable that no account was taken of the additional field due to current in the electrodes. It was found that arc velocity depended upon field and current according to the relation

$$V = cI^{0 \cdot 61}B^{0 \cdot 74} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and it was claimed that this relation was in agreement with theoretical prediction if the arc was considered as a physical conductor experiencing a retarding force proportional to V^2.

This work suffers from the same limitations as that of Winsor and Lee in that the same electrodes were apparently used again and again, and a still camera with a rotating disc photographed the arc only about every 3- or 5 cm along the electrodes. These two factors prevented continuous and discontinuous movement from being distinguished clearly. This is illustrated in Fig. 13(*a*),

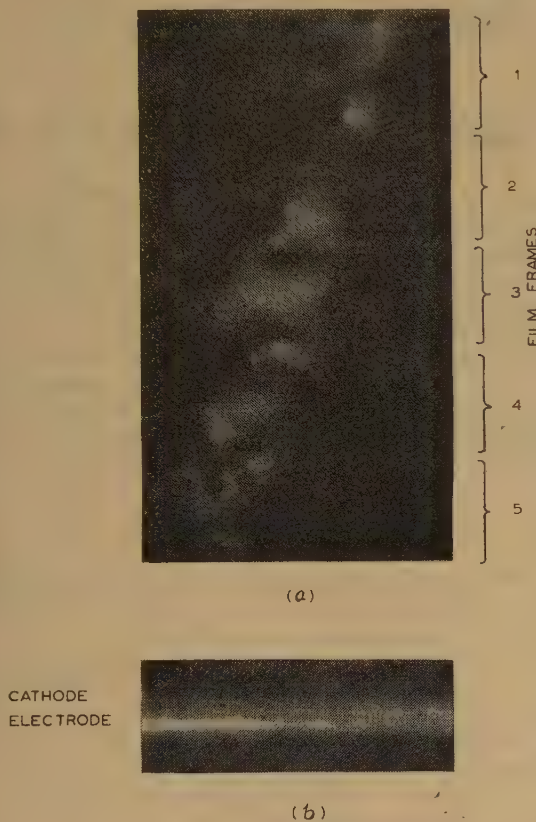

Fig. 13.—Discontinuous motion of a d.c. arc moving between parallel steel electrodes.

which shows a succession of frames taken at 960/sec, where it would appear from the uniform progression of the cathode spot in succeeding frames that the motion might be continuous. There was, in fact, no continuous motion at all on the cathode used in this test, as is shown in Fig. 13(*b*). If such motion were photographed only at about every 3 cm, it would appear from the uniform distance moved between exposures to have been continuous. This may account for the high velocities, up to 50 m/sec, recorded by Eidinger and Rieder.

The increase in velocity with increase in arc current[8] is partly accounted for by the magnetic field of the current flowing in the electrodes (insufficient information is given to allow this to be calculated), and partly by the additional thermal forces on an arc moving vertically. This last factor has been confirmed by photographing arcs moving between vertical electrodes with current feed to each end. The velocities were found to exceed those of horizontally moving arcs and the difference increased with increasing current, being about 50% greater at 60 amp, 120% greater at 180 amp, and 220% greater at 500 amp.

(5) DISCUSSION OF RESULTS

Arc movement in a magnetic field is controlled chiefly by the movement of the cathode spot, which may be continuous or discontinuous.

Continuous movement depends upon magnetic fields at the cathode surface, which may act within the solid or liquid metal or in the cathode-fall region.[5] It is not, therefore, surprising that the movement varies greatly with the cathode material and surface layers, and is independent of column processes.

The fact that the velocity on polished brass electrodes is considerably greater than that on mild-steel electrodes, as shown in Fig. 5, would appear, at first sight, to be in disagreement with some previous results[1] for which current was fed into and out of one end of the electrode system only. The arc then moved under the self-field due to current flow in the electrodes, and the velocity on steel electrodes was much higher than that on brass, for a given arc current. The two apparently conflicting results may be explained in the light of evidence that the seat of arc movement is at the cathode surface.[5] With the externally applied magnetic field, it is shown in Appendix 8.2 that the transverse flux density within a ferromagnetic electrode is approximately twice that in the undisturbed external field, or in a non-magnetic electrode. With the self-field produced by current in the electrodes, the flux density within a ferromagnetic electrode is greater than that in a non-magnetic electrode by a factor of the relative permeability. Thus, the ratio of the fields at the surface of steel and brass electrodes is many times greater for self-field than for external field. Hence the results obtained both in a self-field and in an external field could be explained if the inherent properties of the two metals were such that when the actual fields within their surfaces were equal, the cathode velocity on brass was somewhat greater than that on steel.

If the driving field were operative just outside the electrode surface instead of just inside, then a similar argument could apply, since for self-field, brass and steel would have approximately equal flux densities, but, as shown in Appendix 8.2, a steel electrode in a transverse external field has negligible field immediately outside that part of the surface where the spot travels.

Although it has been shown that the fields causing cathode motion are at the cathode surface, it is not clear whether they operate within solid/liquid metal or in the cathode-fall region in spaces between surface asperities. It has not been possible to decide where the seat of movement is situated, especially since the fields discussed above are modified because there is not an ideal solid/vapour interface in the cathode spot.

There is experimental evidence, at least at reduced pressures, that an arc cathode spot may consist of separate emitting points, all of about the same size and current density.[9] There is some theoretical evidence that a cathode spot on copper or brass may split into two parts at currents of about 40 amp.[10] If multiple spots do form above 40 amp and move independently, this could account for the cathode velocity rising as current increases up to 40 amp, but remaining constant at higher currents.

At the present time, no completely satisfactory theory of electron emission from the cathode has been suggested, although many partially successful hypotheses have been offered. Actually, many processes probably occur simultaneously, with their relative roles a function of many variables, so that it is not possible to suggest a detailed theory of continuous arc cathode movement.

The mechanism of random jumping is by no means clear, but discontinuous motion has been shown to arise from a combination of column and cathode-spot processes. Dunkerley and Schaefer[11] have suggested that the jumping occurs when the arc column is blown forward and ionic bombardment pre-conditions the cathode, so that breakdown from the column to a suitable

site on the cathode can occur. The new arc root then takes over the main arc current. The advance of the arc column undoubtedly plays an important part in the process, since it was found that, when the circuit impedance was made resistive, the arc showed less tendency to move by random jumps, presumably because the arc column could not extend with the same ease as when the circuit was more inductive. As the velocity of random jumping has been found to be dependent on cathode material, it must be assumed that the establishment of the new electron emission site, whether it occurs by ion bombardment or other process, and the transfer of the main discharge, occupy a time which is not negligible compared with that for the overall movement.

Increase in circuit L/R ratio caused an increase in column mobility with a consequent increase of discontinuous velocity of approximately five times. Fig. 9 shows that the velocity can also increase by a factor of four when the material of the cathode is changed but is still non-magnetic. This suggests that the time for column advance is of the same order as the time for forming new emission sites and their taking over the arc current from the previous cathode spot. Since the overall time taken for cathode movement is independent of current, it suggests that both of these processes may take place at a rate independent of current. This could be so for the cathode-root process if multiple-emission sites are established simultaneously and independently.

It is shown in Fig. 11 that, for discontinuous movement, the velocity reached a limiting value for a given flux density. The column near the arc roots nearly always appears, at atmospheric pressure, to exhibit a certain 'stiffness'. As the inter-electrode gap is increased, the restricting effect on the arc column becomes less marked, until at a gap of about 5 cm the centre part of the column is apparently quite unhindered in its motion by the arc roots. It therefore seems possible that the limiting arc velocity attained at a gap of 5 cm corresponds to the maximum rate of deflection of the arc column for a given flux density.

(6) ACKNOWLEDGMENTS

The work was carried out in the Short-Circuit Laboratory at Queen Mary College, University of London. The authors wish to thank Professor M. W. Humphrey Davies for the facilities provided and for his encouragement.

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(8) APPENDICES

(8.1) Calculation of Flux Density at Cathode Surface due to Electrode Currents

The current feed arrangements to the electrode system used by Winsor and Lee are shown in Fig. 1(*d*). For the purposes of calculation, it will be assumed that in the electrodes the arc current is uniformly distributed over the cross-section, so that in deducing magnetic flux densities the current may all be considered to flow along the electrode axis.

At the cathode spot, the current flow in the discharge itself will not produce a transverse flux density if deviation of the column from the normal is small (a pinch force will be operative). Therefore only current flow in the electrodes need be considered. The idealized diagram of the arc is shown in Fig. 14.

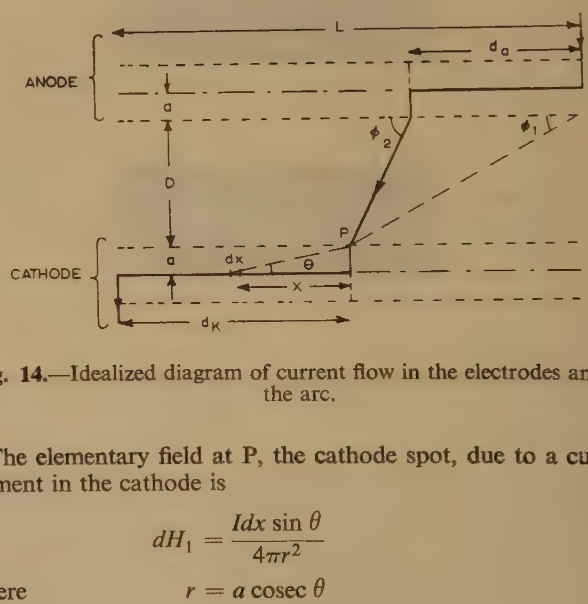

Fig. 14.—Idealized diagram of current flow in the electrodes and in the arc.

The elementary field at P, the cathode spot, due to a current element in the cathode is

$$dH_1 = \frac{Idx \sin\theta}{4\pi r^2}$$

where $r = a \operatorname{cosec}\theta$

and $x = -a \cot\theta$

so that $dx = a \operatorname{cosec}^2\theta d\theta$

Therefore
$$dH_1 = \frac{Ia \operatorname{cosec}^2\theta d\theta \sin\theta}{4\pi a^2 \operatorname{cosec}^2\theta}$$
$$= \frac{I \sin\theta d\theta}{4\pi a}$$

Integrating along all parts of the cathode in which current flows,

$$H_1 = \frac{Id_k}{4\pi a\sqrt{(d_k^2 + a^2)}}$$

Treating the anode current in a similar manner, the field set up at P is

$$H_2 = \frac{I}{4\pi(D+a)}(\cos\phi_1 - \cos\phi_2)$$

$$= \frac{I}{4\pi(D+a)}\left\{\frac{L-d_k}{\sqrt{[(L-d_k)^2+(D+a)^2]}} - \frac{d_a-d_k}{\sqrt{[(d_a-d_k)^2+(D+a)^2]}}\right\}$$

In most cases it is true to say that, averaging over the whole arc motion, $d_k \simeq L - d_a$.

Thus, $$H_2 = \frac{I}{4\pi(D+a)}\left\{\frac{L-d_k}{\sqrt{[(L-d_k)^2+(D+a)^2]}}\right\}$$

The total field at P $= H_1 - H_2$

$$= \frac{I}{4\pi}\left\{\frac{d_k}{a\sqrt{(d_k^2+a^2)}} - \frac{L-d_k}{(D+a)\sqrt{[(L-d_k)^2+(D+a)^2]}}\right\}$$

For gaps and electrode sizes tested, $d_k^2 \gg a^2$ and $(L-d_k)^2 \gg (D+a)^2$. The field at P is therefore simplified to

$$H = \frac{I}{4\pi}\left(\frac{1}{a} - \frac{1}{D+a}\right)$$

The flux density just outside the electrode is

$$B = \frac{\mu_0 I}{4\pi}\left(\frac{1}{a} - \frac{1}{D+a}\right)$$

For the electrode system used by Winsor and Lee,

$$D = a = \tfrac{1}{8}\text{in} = 0{\cdot}00318 \text{ metre}$$

Therefore

$$B = I \times 10^{-7}\left(\frac{1}{0{\cdot}00318} - \frac{1}{2\times 0{\cdot}00318}\right)\text{Wb/m}^2$$

$$= 1{\cdot}57I \times 10^{-5}\,\text{Wb/m}^2 \quad . \; . \; . \; . \; . \; . \; . \quad (3)$$

(8.2) Calculation of Flux Density at the Surface of a Ferromagnetic Electrode in a Transverse Magnetic Field

Two assumptions are made:

(*a*) The normal flux density, B_n, is constant across the gas-metal interface.

(*b*) The tangential magnetizing force, H_t, at any point on the electrode surface is constant across the interface.

Prior to introducing the rod (Fig. 15), and in a field with uniform magnetizing force H_0, the magnetic potential at any point is given by

$$U(r, \phi) = U_0 + H_0 r\cos\phi \quad . \; . \; . \; . \quad (4)$$

(For convenience define $U(r,\phi)$ so that $U_0 = 0$.)

As there is no current source in the field, Laplace's equation for the system is

$$r^2\frac{\partial^2 U}{\partial r^2} + r\frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial\phi^2} = 0 \quad . \; . \; . \; . \quad (5)$$

The variables may be separated by putting $U = R\psi$, where R is a function of r only, and ψ is a function of ϕ only.

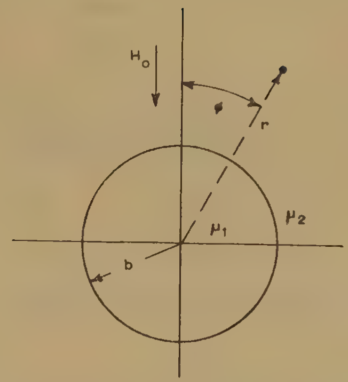

Fig. 15.—Magnetic electrode in a uniform magnetic field.

Substituting in eqn. (5),

$$\frac{r^2}{R}\frac{d^2R}{dr^2} + \frac{r}{R}\frac{dR}{dr} + \frac{1}{\psi}\frac{d^2\psi}{d\phi^2} = 0 \quad . \; . \; . \; . \quad (6)$$

Consider first the functions of r:

$$\frac{r^2}{R}\frac{d^2R}{dr^2} + \frac{r}{R}\frac{dR}{dr} = a^2, \text{ say}$$

Therefore $$R = A_1 r^{\pm a} \quad . \; . \; . \; . \; . \; . \; . \quad (7)$$

where a is an integer.

Considering now the function of ϕ,

$$a^2 + \frac{1}{\psi}\frac{d^2\psi}{d\phi^2} = 0$$

The solution of this is

$$\psi = A_2\cos a\phi + A_3\sin a\phi \quad . \; . \; . \; . \quad (8)$$

For U to vary as in eqn. (4) at large values of r, where the cylinder's presence has little effect, requires that A_3 shall be zero.

Thus, $$U = \sum_{a=0}^{\infty} B_a r^a\cos a\phi + \sum_{a=0}^{\infty} C_a r^{-a}\cos a\phi$$

where B_a and C_a are the new constants.

At large values of r, $U_e = H_0 r\cos\phi = B_1 r\cos\phi$. Hence, $H_0 = B_1$ and all other values of $B_a = 0$.

Thus, $$U_e = H_0 r\cos\phi + \sum_{a=0}^{\infty} C_a r^{-a}\cos a\phi \quad . \; . \; . \quad (9)$$

Inside the electrode,

$$U_i = \sum_{a=0}^{\infty} D_a r^a\cos a\phi + \sum_{a=0}^{\infty} F_a r^{-a}\cos a\phi$$

Since U_i is finite in the rod, all values of F_a must be zero, except F_0.

$$U_i = \sum_{a=0}^{\infty} D_a r^a\cos a\phi + F_0$$

As B_n is continuous across the metal-gas interface,

$$-\mu_2\frac{\partial U_e}{\partial r} = -\mu_1\frac{\partial U_i}{\partial r}$$

Now the values of C_1, D_1, C_2, D_2, etc., are fixed, whatever the value of ϕ:

$$\mu_2 H_0 = \mu_2 C_1/b^2 + \mu_1 D_1 \quad . \; . \; . \; . \; . \; . \quad (10)$$

Since H_t is continuous across the interface,

$$-\frac{1}{r}\frac{\partial U_e}{\partial \phi} = -\frac{1}{r}\frac{\partial U_i}{\partial \phi}$$

Thus, $$H_0 \sin \phi b = (D_1 b - C_1/b) \sin \phi \quad . \quad . \quad . \quad (11)$$

and D_2, C_2, D_3, C_3, etc., are all zero.

Combining eqns. (10) and (11),

$$C_1/b^2 = D_1 - H_0$$

$$\mu_2 H_0 = \mu_2(D_1 - H_0) + \mu_1 D_1$$

$$D_1 = \frac{2\mu_2 H_0}{\mu_2 + \mu_1}$$

$$C_1 = \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} H_0 b^2$$

It is required to find B_t at the point just inside the rod, where $\phi = \pi/2$.

$$U_1 = D_1 r \cos \phi + F_0$$

The transverse flux density inside the cylinder (for $0 < r < b$) is

$$B_{ti} = -\frac{\partial U_i}{r\partial \phi}\mu_1$$

$$= \frac{2\mu_2 H_0}{\mu_2 + \mu_1}\mu_1$$

$$\simeq 2H_0\mu_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The transverse flux density at the cylinder surface and external to it (for $r = b$) is

$$B_{to} = -\frac{\partial U_e}{r\partial \phi}\mu_2$$

$$= \frac{2\mu_2}{\mu_2 + \mu_1} H_0 \mu_2$$

$$\simeq 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Away from the surface of the electrode,

$$U_e = H_0 \cos \phi + \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} H_0 b^2 \frac{1}{r} \cos \phi$$

Hence $$B_t = \mu_2 H_0 \sin \phi + \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} H_0 \mu_2 b^2 \frac{1}{r^2} \sin \phi$$

$$\simeq H_0 \mu_2 \sin \phi\left(1 - \frac{b^2}{r^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

621.389 : 629.135

The Institution of Electrical Engineers
Paper No. 2912 U
Aug. 1959

ELECTRICAL MATERIALS AND COMPONENTS FOR AIRCRAFT POWER EQUIPMENT OPERATING AT HIGH TEMPERATURES

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(*The paper was first received* 2nd *December*, 1958, *and in revised form* 3rd *February*, 1959.)

SUMMARY

The paper describes the materials now available and in course of development which are suitable for use in components for operation at high ambient temperatures. The present and future developments of components for aircraft electrical power distribution are described.

(1) INTRODUCTION

With the introduction of faster aircraft, higher ambient temperatures will be encountered, and it is highly desirable that electrical equipment should be capable of operating in these ambient temperatures, as it is impracticable to concentrate all the components in a few areas that can be cooled.

Present electrical accessories, in general, are suitable for functioning in an ambient temperature of 70° C and may have hot-spot temperatures up to about 130° C. The development of most equipment for use without cooling in an ambient temperature of 150° C is possible. There are isolated items, however, such as rectifiers, which are limited to a maximum operating temperature below 100° C. The development of equipment to work in an ambient temperature of 200° C with hot-spot temperatures between 250 and 300° C is more remote as no known organic materials will withstand these temperatures for any length of time. Inorganic materials, where suitable, can be used but there is a weight penalty to pay.

Aircraft equipment has a relatively short life compared, for example, with power station plants and consequently it is normal practice to operate materials at considerably higher temperatures.

(2) MATERIALS

(2.1) Insulating Materials

Insulating materials of the rigid types for use at 300° C are available in the form of ceramics and glass-bonded mica. For equipment where the temperature does not exceed 200° C there are several materials available such as alkyd and melamine resins, modified phenolic resins, glass-reinforced epoxy and polyester resins and silicone moulding materials. Care must be exercised in the use of phenolic resins as they tend to track.

High-temperature glasses suitable for mechanical use at about 700° C exist. They contain a high percentage of pure silica, have a specific gravity of about 2·2, coefficient of linear expansion of about 8×10^{-7} per deg C and volume resistivity about 10^9 ohm-cm at 350° C. These values may be improved upon by the use of pure fused silica, but the material cannot be readily moulded into intricate shapes.

A ceramic recently developed in the United States known as Pyroceram is suitable mechanically for use up to about 750° C. This material is tough, has virtually no flexibility but can be worked using glass techniques. The dielectric constant is approximately 5·5, the loss angle 0·002 at 10^6 c/s and resistivity at 350° C about 10^8 ohm-cm.

Development work is progressing in this country on non-oxide ceramic dielectrics such as boron nitride, silicon nitride and aluminium nitride, some of which may have special electrical applications.

Glass-bonded mica materials are available for use to about 300° C, being limited largely by the grade of glass. A more stable variety of this type of insulation is available in the United States, where a synthetic mica is used in place of the natural mica powder used in Great Britain. The stability is achieved by substituting a fluorine group in the mica molecule in place of the hydroxyl group found in natural mica.

Where flexibility is required, silicone rubbers are the only suitable materials for use at 200° C, and for short periods they will withstand 250° C. The silicone rubbers are, however, attacked by kerosene. Recently in the United States, a solvent-resistant fluorosilicone rubber has been developed, and, although it is still too early to say that this material will be suitable for electrical insulation on aircraft equipment, it is interesting to know that some advances have been made in obtaining silicone rubbers resistant to the wide range of fluids encountered on an aircraft.

Polytetrafluoroethylene (p.t.f.e.) and polychlorotrifluoroethylene (p.c.t.f.e.) are less flexible than silicone rubber. A copolymer of the latter material with vinylidene fluoride is now available in experimental quantities in the United States as an elastomer, and is known as Kel-F elastomer. From limited information available it would not appear to be suitable above 150° C. The electric strength lies between 500 and 600 volts per mil, the volume resistivity is about 10^{14} ohm-cm, the dielectric constant at 1 Mc/s about 4 and the loss angle at the same frequency about 0·12. It is ozone resisting and does not support combustion. P.t.f.e. (Fluon, Teflon), however, is suitable for continuous operation at 200° C and, in certain applications where physical pressure is not too great, may be used at 250° C.

Synthetic rubbers of the polychloroprene (Neoprene) and butadiene acrylo-nitrile (Hycar or Paracril) types are unsuitable for continuous use at 150° C, although they will withstand this temperature for short periods. The latter type, although possessing good oil-resistance properties, does support combustion comparable with normal grades of natural rubber.

In the United States elastomers of the polyacrylic type, polyperfluorobutyl acrylate (Poly F.B.A.), are being used in a few applications as seals and gaskets for use up to 175° C for short periods, but at 150° C a reasonable life is obtained. They possess good resistance to ester-base fluids.

Progress is also being made in the United States in modifying the polysulphide rubbers (Thiokols) to allow their use up to 175° C with some prospects of extending the limit to about 260° C.

A recent development in the United States to obtain an elastomer for high-temperature applications has resulted in a material, perfluoropropylene vinylidene fluoride copolymer, which, from American information, will withstand 300° C. This material is known as Viton A. Elastomeric fluoropolyesters have also been developed recently in the United States but their temperature limit is about 200° C.

Written contributions on papers published without being read at meetings are invited for consideration with a view to publication.

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Although these elastomers have been included under the heading of insulating materials, the electrical properties are, in many cases, poor at normal temperatures with a further decrease at high temperatures.

Fabrics in the form of tapes and cloths are available to cover a wide temperature range. For use up to 170° C, Terylene fabric impregnated with a modified phenolic varnish is satisfactory, and for temperatures up to 200° C glass cloth impregnated with silicone varnish or coated with silicone elastomer can be used. Varnish-bonded glass-backed mica tape may also be used up to this temperature. Unimpregnated glass fabric may be employed at 300° C. It is possible to impregnate the glass fabric with p.t.f.e. dispersion, but the limiting temperature is then about 250° C; above 300° C little choice remains, apart from mica and asbestos.

Good-quality phlogopite mica can be used up to 750° C but it is relatively inflexible. Asbestos cloths and tapes, on the other hand, are flexible but very weak mechanically and require to be protected against moisture at normal temperatures. For the interleaving of coils, the use of anodized aluminium foil is a possibility.

It has been found that the operating temperature of polythene can be raised by irradiation of the material by electron bombardment provided that adequate anti-oxidant is present. In the case of equipment wire, for example, preliminary tests show that it may be possible to use this type of wire continuously at 130° C, for 100 h at 150° C, for 7 h at 200° C or 1 h at 250° C. Although only limited experience is at present available, polythene which has been irradiated may prove to be a useful intermediate-temperature material where relatively low electrical losses are required and the cost of p.t.f.e. is not justified.

(2.2) Conducting Materials

The most commonly used conducting material is copper, which at high temperatures must be protected against oxidation. A coating of tin or cadmium is suitable up to 150° C, but for higher temperatures a heavy coating of nickel is required which, from limited experience, would appear to be effective up to 400° C. Nickel, being a magnetic material, can influence the effective resistance of a wire, but, provided that the thickness of the nickel does not exceed 10% of the radius of the conductor, no appreciable difference in the resistance of a wire under d.c. and a.c. conditions up to 3 kc/s is detectable. Above this frequency the increase in resistance may be appreciable, and the use of either a silver-plated or a solid-silver wire may become necessary, although at temperatures in excess of 200° C oxidation may occur. The use of gold or gold-plated conductors may be the only solution for the higher temperatures and frequencies.

Aluminium and its alloys are unsuitable in general for use in electrical equipment above 200° C owing to loss of mechanical strength.

(2.3) Magnetic Materials

No trouble is expected in the use of soft iron for magnetic purposes up to 250° C, but the properties of certain of the nickel irons, particularly those with low Curie points, may be affected at higher temperatures.

Table 1 gives an indication of the Curie points for a number of magnetic materials.

For 4-mil grain-oriented silicon-iron at a core temperature of 224° C and at a flux density of 16 300 gauss, the core loss is about 10% less and the exciting current about 25% greater than at 30° C.

(2.4) Spring Materials

For springs for use up to 250° C the use of a high-content chromium ferrous alloy is recommended, but where the temperature does not exceed 150° C beryllium-copper springs may be suitable except where very heavy stressing is involved. Nickel-chromium springs of the Inconel or Nimonic types may be used at temperatures above 250° C.

Table 1

Curie Points for Various Magnetic Materials

Material	Curie point
	deg C
2 % V-Permandur	980
Soft iron	770
0·5 % Silicon-iron	760
4 % Silicon-iron	690
Radiometal	520
Mumetal	390

(2.5) Winding Wires

The temperature limits referred to in this Section are higher than those recommended in general engineering practice as given in B.S. 2757, since they are based on the relatively short life of aircraft equipment compared with the life expected from commercial machines and apparatus.

Fibrous-covered winding wires using cotton, silk, rayon and paper and oleo-resinous enamelled wires are unsuitable above 120° C. Modified polyamide (nylon) enamelled wires may be used up to 130° C, vinyl-acetal enamelled wires to 140° C and epoxy and polyurethane enamelled wires to 150° C. Polyamide and polyester (Terylene) fibrous-covered wires can be used at 130° C and 170° C, respectively, but, owing to the poor space factor, they are rarely used. Fibrous-glass or asbestos-covered wires are preferred, and, when treated with varnish, can be handled satisfactorily provided that reasonable care is used. The temperature limit is governed by the varnish and may be as high as 200° C with silicones.

Since fibrous-covered wires are undesirable because of their poor space factor, work has been directed towards the application of resins to a wire in order to obtain a space factor similar to that obtained with conventional enamelled wires.

Silicone enamel has been successfully applied to wire and, although slightly inferior in regard to abrasion resistance and flexibility compared with the vinyl-acetal type, may be used up to 200° C in coils impregnated with silicone varnish. To improve the abrasion resistance and flexibility, silicone varnishes modified with epoxy and similar resins have been developed, but the operating temperature falls to about 180° C.

Polyester-enamelled wire is now available in this country and possesses both toughness and flexibility, and being thermo-set the coating does not tend to soften at elevated temperatures. This material is promising for use at 180° C and is therefore almost comparable with the silicone-coated wire. It has, in addition, the toughness and flexibility of the vinyl-acetal class.

Winding wires using p.t.f.e. have been in existence for some considerable time, but such a coating does not have adequate resistance to flow, particularly at high temperatures under such pressure as is encountered in a winding. Winding wires, therefore, based on p.t.f.e. alone have not been successful. A wire with a p.t.f.e. coating reinforced with glass fibre, either as a braid or roving, is satisfactory for temperatures up to 250° C, but the disadvantages are twofold, namely the space factor is poor and the winding cannot be impregnated satisfactorily. This wire may be used if the size of the coil can be tolerated and if moisture at normal temperatures can be kept from the winding, e.g. by hermetic sealing with the leads brought through glass-metal seals.

A wire coated with a ceramic-loaded p.t.f.e. enamel, known as

Ceroc T, has been available for some considerable time in the United States, but it appears that this enamel tends to flake off the wire slightly when a coil is being wound. This coating has, however, been used successfully at 300° C in small power transformers in conjunction with nickel-plated copper wire.

Anodized-aluminium wire has been used as a winding wire but the anodizing tends to be porous. This can be improved, however, by passing the wire through a bath of p.t.f.e. dispersion, drying and finally sintering the p.t.f.e. This wire is satisfactory at 250° C and has a better space factor than the glass-reinforced p.t.f.e. wire. No suitable impregnant is, however, available. A further limitation of this wire is the high resistance of the aluminium. To overcome this, an aluminium-clad copper wire has been developed, the aluminium skin of which is anodized. If the electric strength of the film is inadequate, this can be increased by coating with p.t.f.e. dispersion, but, again, the p.t.f.e. limits the temperature to 250° C. Without the p.t.f.e. it may be possible to reach 300° C.

On aircraft, the impregnation of windings is most desirable because of vibration and the presence of moisture at the lower temperatures. No organic impregnating medium will withstand 300° C, and therefore those inorganic glasses which melt at relatively low temperatures are being considered. A glass with a relatively low melting point has been found to offer reasonable prospects, but considerable work remains to be done in finding materials which are compatible, particularly in regard to temperature coefficient.

Recent work has shown that it may be possible to anodize copper and produce a satisfactory insulating film. If this process can be applied to a winding wire, it may be feasible to use it at 400° C. The conventional oxide-film wire has been considered, but, unless the winding is completely sealed against moisture at the lower temperatures, corrosion occurs and complete breakdown of the film results.

A winding wire is in the course of development in the United States using synthetic mica bonded with glass. The wire is coated with this material and partially fired to produce a reasonably flexible wire for the winding operation. After the coil is made, it is heated to the sintering temperature after which the assembly becomes rigid. The operating temperature of this insulation is about 500° C, but owing to the high sintering temperature required, only stainless-steel wire can be used. It is understood that work is in progress to reduce this sintering temperature so that a nickel-clad copper wire with its higher conductivity can be employed. There has, however, been a certain amount of trouble experienced due to the difference in thermal expansion between the copper and the mica, and other fillers, such as alumina, zircon and zirconia, are being investigated in the United States. The glass used must have an extremely low alkali content, which unfortunately coincides with a higher melting point with its inherent oxidation difficulties with copper. The resistivity of such a glass, although adequate, falls from about 10^9 ohm-cm at room temperature to 10^6 ohm-cm at 300° C and 10^4 ohm-cm at 500° C.

(2.6) Slot and Conductor Insulation

For slot and conductor insulation for machines, the most suitable materials are glass fibre and mica treated where necessary with polyester or silicone resins. Where, however, good bond strength is required, as in a rotating part, a modified phenolic-resin varnish would appear at present to be the most suitable. It is the bonding agent for the glass fibre and mica which sets the upper temperature limit of operation.

Pure fused silica has excellent electric strength and resistivity at high temperatures but is too fragile for use in a rotating machine. Fused silica is now also available in fibre form and in certain applications might be tamped into the space between the conductor and the iron and sealed in position by a suitable binder such as an aqueous suspension of calcium sulphate and then baked at about 300° C to form a consolidated block. A composite insulation of this type requires considerable space, and, owing to its relatively low thermal conductivity, machine losses become high. This insulation could be used only in stators because of its poor bond strength. The insulation resistance is good, being about 10^{11} ohm-cm at room temperature and falling to 10^9 at 500° C.

Several types of alumina cements have been considered but none appear to be very promising for machine insulation. Pure alumina cement, which requires to be fired at about 1 800° C, is obviously unsuitable in the presence of copper conductors. The lower-temperature-hardening alumina cements are bonded together with organic materials, and on heating above 300° C the bonding material burns leaving an unfired cement which is too weak mechanically for practical purposes. Ciment Fondu aluminous cement does not suffer from the loss of binding agent as does the pure alumina cement, but on setting it tends to shrink and crack and therefore is of little value for machine insulation. It is possible to add a little graded alumina to increase the mechanical strength but this does not overcome the shrinkage problem. In conjunction with ceramic beads, it might be possible to use this cement for coil-end insulation. In general, cements are unsuitable for slot insulation, since uniform thickness is difficult to achieve without pre-forming. Also it is difficult to obtain sufficiently thin sections of insulation, and therefore only one or at most two conductors per slot can be accommodated unless the machine becomes of unreasonable dimensions. The electrical properties of the cements are greatly inferior to those of ceramics and silica.

A process for the flame spraying of alumina on to metals has been developed, but after exposure to high temperatures oxidation of copper occurs owing to the porosity of the alumina, resulting in its flaking. The plating of the copper with a non-oxidizing metal would appear to be a solution, but experimental results do not confirm this. The spraying of the alumina directly on to the iron in the slot has been successful although the actual application to a small slot is difficult.

The use of ceramic tubes for slot insulation in conjunction with silica fibre has not proved to be very successful owing to the relatively weak mechanical properties of the ceramic. Sleeves manufactured from sintered aluminium oxide are now available with wall thicknesses as small as 0·025 in, and are a practical proposition as a slot liner since they enable several turns per slot to be used. This material has excellent electrical properties at elevated temperatures and good thermal conductivity. For machines for aircraft use, however, it would be necessary to have well-fitting wires and additional packing, possibly in the form of bonded fibrous silica, to prevent damage due to vibration.

Steatite with a high proportion of magnesium silicate may be of use in some instances, although it is weaker mechanically than sintered aluminium oxide and cannot therefore be obtained as thin tubes. This material is extensively used for bead insulators.

Compressed magnesium-oxide components may be suitable where it is possible to apply adequate pressure to compact the material and sufficient protection can be given to retain the compacted material in position. Mineral-insulated cables, which use this material, do not appear to be practical for incorporation in a machine owing to the excessive space occupied.

(2.7) Lubricants

Considerable research work is in progress to develop high-temperature lubricants. Present greases tend to harden after

long periods at 150° C and are sparingly miscible in hydraulic fluids of the mineral and ester-base types. There is, moreover, a tendency to corrode copper.

Experience with molybdenum disulphide, as a powder, in a volatile medium, or as a colloidal dispersion in water or in other liquids, has been disappointing. Molybdenum disulphide has been added to mineral-oil greases but the upper temperature limit for continuous use is governed by the mineral grease. The molybdenum disulphide may provide, however, some safeguard against seizure.

Silicone greases, to which has been added molybdenum disulphide, exist but do not appear to be satisfactory. Silicone greases without additives appear to give a reasonable performance up to 150° C when lightly loaded. Precautions must be taken to prevent contamination from hydraulic fluids when any silicone greases are used.

Considerable work on fluorocarbon lubricants is being done but more work is required before a suitable high-temperature lubricant is available. For very low speeds and light loads, p.t.f.e. can be used as a bearing and can be run dry.

(3) COMPONENTS

This Section summarizes the present position in the development of high-temperature electrical accessories used for the general services on aircraft.

(3.1) Cables

General-service cables insulated with silicone rubber, polyethylene terephthalate and glass for continuous operation at 150° C are now available in a comprehensive range of sizes as single-core cables with copper and aluminium conductors. They are known as Tersil cables. These cables with copper conductors are also suitable for use in essential circuits which are required to function during or after a fire.

For continuous operation at 200° C, there is also a comprehensive range of single-core cables insulated with p.t.f.e. and glass, known as Efglas cables. These are not suitable for use in circuits which must function during or after a fire, but a cable of modified construction containing, in addition, a small quantity of asbestos is available for these services where the ambient temperature for normal operation exceeds 150° C but not 200° C.

For the internal wiring of equipment, comprehensive ranges of p.t.f.e.-insulated wires in single and bi-colours are now available for use where the conductor temperature does not exceed 250° C.

A range of multi-core cables insulated with p.t.f.e. and suitable for continuous use at 200° C for the interconnection of electronic and associated equipment is also available.

Thermocouple cables with the insulation construction of the Tersil and Efglas cables are being developed. Since there is no heating of the conductor due to the flow of current, these cables can be operated at the temperature limit of the insulation, which is 190° C for Tersil and 240° C for Efglas.

Experimental cables have been made for continuous use at 400° C using compressed magnesium-oxide dielectric contained in a copper sheath which is heavily nickel-plated.

(3.2) Cable Terminations

Standard crimped connections employing copper lugs and copper cables appear to be suitable for continuous use up to 200° C but only for short periods above this value. At 200° C, the copper must, however, be protected against oxidation by plating, preferably with nickel. Above 200° C joints must be welded. Tests of crimped connectors on aluminium cables with aluminium lugs have shown that at temperatures above 100° C excessive voltage drops develop, and, in the light of present information, the use of these terminations above 100° C is not recommended. The lugs can, of course, be welded, but this process during maintenance is not possible in an aircraft.

The termination of the mineral insulated cable has been achieved successfully by the use of an alumina insulator brazed to the copper sheath. This cable must, however, be used in conjunction with a terminal block in which a screw and washer or nut and stud is used.

(3.3) Capacitors

Metallized paper and titanate (high 'k') capacitors can be used successfully at 100° C. The tantalum capacitor in a unit of 70 volts 55 μF can be assembled in multiples to cover a wide voltage and capacitance range for use up to 125° C, although it is possible to operate this capacitor up to 150° C for short periods. The polyester-film capacitor is now available for use up to 130° C. The vitreous-enamel capacitor which has been developed in the United Kingdom is suitable for operation at 150° C and can be used as a replacement for small-capacitance mica capacitors.

For higher temperatures, the glass capacitor is satisfactory at 200° C, but the capacitance per unit volume is small. The silvered-mica capacitor can be used up to 300° C provided that the associated materials will withstand this temperature. P.t.f.e. has been tried as a high-temperature capacitor dielectric but with little success owing to the dimensional instability of the p.t.f.e. film. Also, the capacitance per unit volume is low owing to the low permittivity of the material.

(3.4) Fuse Links and Holders

Fuse links and holders are available for continuous use at 200° C.

(3.5) Meters

As meters will almost invariably be installed in the crew compartment, existing designs will be suitable. No steps are being taken, therefore, to design a range of high-temperature meters.

(3.6) Plugs and Sockets

Standard aircraft plugs and sockets with phenolic moulded insulation and aluminium shells can be used at a maximum temperature of 90° C and with alkyd mouldings up to 150° C. More recent developments, however, with aluminium-alloy shells and silicone-rubber insulation have enabled a range of plugs and sockets to be made which will withstand a maximum temperature of 190° C. Plugs and sockets have been developed for higher temperatures for special applications employing glass-metal seals for the pin insulation and steel shells. P.t.f.e. has been used for the insulation of plugs and sockets in some American designs but problems exist with pin location.

(3.7) Rectifiers

At present, selenium rectifiers are suitable at 85° C but no rectifiers are in large-scale production for use above this temperature. Development, however, of the silicon rectifier is progressing, and limited supplies are available for use at 100° C. There is a reasonable prospect that the silicon rectifier may be developed for use at 200° C provided that ancillary parts are available.

The germanium rectifier is not very suitable above 55° C although if lightly loaded an output at 85° C can be obtained. The tellurium rectifier may be made to function up to 400° C but development is far from complete.

(3.8) Resistors

Resistors for use above 100° C are available in several forms. Cracked carbon may be run at a body temperature of 150° C and gold-platinum film up to 200° C. Vitreous wire-wound resistors can operate up to 300° C. Development work is in progress to increase the working temperature of the gold-platinum and tin-antimony oxide film resistors to 400° C. It should be noted that the temperatures quoted above for resistors are maximum body temperatures, the permissible ambient temperature being determined by the loading selected.

(3.9) Switchgear and Relays

The development of switchgear and relays for use at ambient temperatures of 150° C presents few difficulties with regard to basic materials, but new designs are required in order to obtain the desired characteristics. Greater use of latched gear may be necessary in order to meet the requirements for pull-in voltage.

Switchgear and relays to work in ambient temperatures of 200° C present problems of materials as well as operation. This gear will almost certainly be heavier and bulkier than existing units and their availability is considerably distant.

(3.10) Terminal Blocks

Since there is little temperature rise in a terminal block due to the flow of current, no difficulty should arise in the production of this item for use in an ambient temperature of 200° C. Several materials are available, although glass-bonded mica is probably the most suitable. For higher temperatures, moulded alumina can be used.

(3.11) Transformers

By using existing materials, transformers have been made for continuous use at an ambient temperature of 120° C with hot-spot temperatures in the windings of 250° C. These transformers employ cold-rolled grain-oriented silicon-iron laminations 0·004 in thick and have close-fitting shrouds to form heat-dissipating surfaces. An additional feature is an inner thermal conductor of soft aluminium which is introduced between the primary and secondary windings and assists in reducing the thermal gradient across the coil.

The formers used for the windings are made from aluminium and assembled in two halves insulated from each other to avoid a short-circuited turn. The formers are insulated with silicone-bonded glass fabric in conjunction with small pieces of mica to give added protection at corners. The winding wires are insulated with double glass-roving silicone-impregnated varnish, and the complete winding is treated with silicone-impregnated varnish and also with a silicone elastomer to make contact between the windings and the external case to reduce the thermal resistance. Lead-out wires are insulated with glass-silicone-impregnated sleeving.

Silicone elastomers have been used successfully as an impregnating medium but care must be exercised in using this material to prevent voids. Careful control of curing time and temperature are necessary. It has been possible to reduce the temperature rise of a winding to about 80° C compared with about 120° C for conventional transformers.

(4) CONCLUSIONS

The development of basic materials for high-temperature use in electrical equipment is being pursued actively by many investigators, but the development of the equipment for ambient temperatures of 200° C and higher is progressing slowly. There is no doubt that isolated items will be required, but the need for a complete electrical power system to operate at an ambient temperature of 200° C is somewhat obscure at present. There is, however, little doubt that a need will exist for a complete system for use at 150° C, and effort should be directed towards this achievement.

(5) ACKNOWLEDGMENT

Acknowledgment is made to the Controller of H.M. Stationery Office for permission to publish this paper.

621.313.33

The Institution of Electrical Engineers
Paper No. 3049 U
Aug. 1959
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THE STARTING OF SINGLE-PHASE INDUCTION MOTORS HAVING ASYMMETRICAL STATOR WINDINGS IN QUADRATURE

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(*The paper was first received* 5*th February, and in revised form* 31*st March*, 1959.)

SUMMARY

The operation of an asymmetrical 2-phase induction motor on a single-phase supply system is analysed by the method of symmetrical components, and performance equations for the special problem of starting the machine are expressed in terms of three dimensionless parameters. The effects of variation of these parameters are studied in detail, and conditions for optimum starting performance are established. Starting characteristics are given for a wide range of the parameters, but it is shown that the best performance is obtained with a capacitive convertor selected to give minimum unbalance. Properly interpreted, the results are applicable to standard single-phase induction motors of any rating.

LIST OF SYMBOLS

I_a, I_{ab} = Current in the auxiliary winding under single-phase and balanced 2-phase conditions, respectively.
I_m = Current in the main winding.
I, I_b = Supply current under single-phase and balanced 2-phase conditions, respectively.
I_1, I_2 = Positive- and negative-sequence components, respectively, of the current I_m.
k = Ratio of the effective number of turns of the auxiliary winding to that of the main winding.
Q_1, Q_2, Q_3 = Starting qualities defined as the respective ratios of single-phase to balanced 2-phase values of the starting torque per ampere of supply current, the starting torque per ampere squared of supply current, and the starting torque per watt of stator copper loss.
R_a, R_m = Resistances of the auxiliary and the main windings, respectively.
T, T_b = Starting torque under single-phase and balanced 2-phase conditions, respectively.
U = Unbalance factor, defined as the ratio $|I_2/I_1|$.
V = Supply voltage.
V_a, V_m = Voltages across the auxiliary and main windings, respectively.
V_1, V_2 = Positive- and negative-sequence components, respectively, of the voltage V_m.
V_Y = Voltage across the phase convertor.
Y = Total admittance of the phase convertor.
Y_{ext} = Admittance of the external convertor.
Y_s = Input admittance of the main winding of the motor at standstill.
Y_1, Y_2 = Input admittance of the main winding to positive- and negative-sequence currents, respectively.
y = Dimensionless parameter defined as $|Y_s/Y|$.
$\alpha = \beta - \phi$
β = Argument of the admittance Y.
ϕ = Argument of the admittance Y_s.

Written contributions on papers published without being read at meetings are invited for consideration with a view to publication.
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(1) INTRODUCTION

Most single-phase induction motors, with the exception of the shaded-pole motor with which the paper is not concerned, employ a stator having two windings in space quadrature usually with different numbers of turns. One of the windings, called the 'auxiliary' winding, is designed to produce, either by itself or with the assistance of an external series impedance, a resistance/reactance ratio which differs considerably from that of the main winding. When the two windings are connected in parallel across the single-phase supply, the currents flowing in them differ in time phase and hence produce a rotating field. In split-phase and capacitor-start motors, the auxiliary winding is used only for starting and is disconnected once the motor has reached sufficient speed. In capacitor-start-and-run motors, however, this winding, with appropriate series capacitors, is used both for starting and normal running of the machine. Care is taken in the design of the winding and the choice of the running capacitor to simulate as near balanced 2-phase operation of the machine under load as possible.

Single-phase motors can, in general, be considered as special cases of unbalanced operation of asymmetrical 2-phase motors, and, as demonstrated by Suhr,[1] the theory of symmetrical components can be used to predict their performance with great ease. The device used to introduce a phase difference in the currents of the two windings can be treated as a static phase convertor which effectively converts the single-phase supply voltage to a 2-phase 'air-gap voltage', not necessarily balanced. The problems of operation of the motor then resolve into the choice of proper convertor impedance to give satisfactory starting, run-up and full-load performance.

The single-phase induction motor is widely used and an extensive literature on the theory of its operation and, in particular, on the design of starting windings[2,3,4,5] has grown up. Of previous authors, however, only Suhr[1] and Jordan and Lax[5] have given generalized theoretical treatment to cover most types of single-phase motors. Suhr's excellent paper[1] deals with the general operation of the machine but does not cover the problem of starting in any great detail. Jordan and Lax[5] discuss specifically the problem of the starting of single-phase machines, and, in parts, their approach is similar to that of the present paper, but owing to what appears to be an error in their theoretical treatment they have arrived at results considerably different from those derived here.

A good method of evaluating the single-phase operation of a polyphase induction motor is to compare its performance with that of the machine under balanced polyphase operation. In an earlier paper[6] this method was utilized in an analysis of the starting of a 3-phase motor connected to a single-phase supply, and it was shown that the performance equations can be expressed in terms of two dimensionless parameters. The present paper is an extension of the method to cover the starting of asymmetrical 2-phase induction machines from a single-phase supply, and it is shown that the equations in this case contain three dimensionless parameters. Since different types of starting only change the numerical values of the parameters, it is a

simple matter to compare their characteristics and hence devise suitable design criteria for the motor.

(2) THEORY

(2.1) General

In Fig. 1, m and a represent, respectively, the main and the auxiliary stator windings of an asymmetrical 2-phase motor

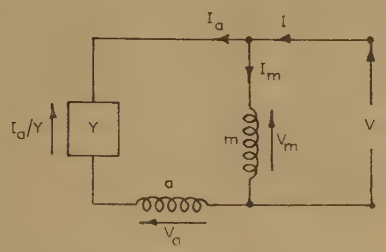

Fig. 1.—Stator windings of an asymmetrical 2-phase induction motor connected to a single-phase supply with a static phase convertor in circuit.

connected to a single-phase supply with a static phase convertor of admittance Y in circuit. The auxiliary winding has k times as many turns as the main winding and ideally would have $1/k^2$ times as much leakage admittance. For analytical purposes it will be assumed that any difference of the actual leakage admittance of the auxiliary winding from its ideal value is included in the convertor admittance Y.

From the application of Kirchhoff's laws, inspection equations for this circuit are as follows:

$$I = I_m + I_a \quad (1)$$

and

$$V = V_m = V_a + I_a/Y \quad (2)$$

These equations can be solved in terms of symmetrical component theory as applied to asymmetrical 2-phase machines,[1] by utilizing the following substitutions:

$$V_m = V_1 + V_2 \quad (3)$$

$$V_a = jkV_1 - jkV_2 \quad (4)$$

$$I_m = V_1Y_1 + V_2Y_2 \quad (5)$$

and

$$I_a = j\frac{V_1Y_1}{k} - j\frac{V_2Y_2}{k} \quad (6)$$

where V_1 and V_2 are, respectively, the positive- and negative-sequence components of the voltage V_m, and Y_1 and Y_2 are the respective values of the input admittance of the main winding to positive- and negative-sequence currents.

The solutions of eqns. (1) and (2) are given by

$$V_1 = \frac{V[Y_2 + k(k-j)Y]}{Y_1 + Y_2 + 2k^2Y} \quad (7)$$

$$V_2 = \frac{V[Y_1 + k(k+j)Y]}{Y_1 + Y_2 + 2k^2Y} \quad (8)$$

from which

$$I_m = \frac{V[2Y_1Y_2 + k(k-j)YY_1 + k(k+j)YY_2]}{Y_1 + Y_2 + 2k^2Y} \quad (9)$$

$$I_a = \frac{V[Y_1Y(1+jk) + Y_2Y(1-jk)]}{Y_1 + Y_2 + 2k^2Y} \quad (10)$$

and the supply current $I = I_m + I_a$ is given by

$$I = \frac{V[2Y_1Y_2 + (1+k^2)Y(Y_1+Y_2)]}{Y_1 + Y_2 + 2k^2Y} \quad (11)$$

From eqns. (7), (8), (9), (10) and (11), the performance of the machine for any given values of k and Y can be predetermined.

These equations are very general in their nature, and with proper interpretation of the convertor admittance Y, they can be used to calculate the complete performance of most single-phase induction motors. In the case of self-starting split-phase motors, Y is the difference $(Y_a - Y_m/k^2)$, where Y_a and Y_m are, respectively, the input admittances of the auxiliary and the main windings; while in resistance-start, reactance-start, capacitor-start and capacitor-start-and-run motors, the external admittance, Y_{ext}, used in series with the auxiliary winding is included in Y, i.e. $Y = Y_a - Y_m/k^2 + Y_{ext}$.

The general characteristics of single-phase motors, and even their names, depend almost entirely upon the nature of the convertor admittance, and before discussing in detail the starting performance of motors with various types of phase convertors it is desirable to consider briefly the performance required of the phase convertor.

For motors where the auxiliary winding is in circuit under normal running conditions, economic considerations require the phase convertor to produce on load as near balanced 2-phase operation as possible. The admittance of such a phase-convertor, however, is usually too small to produce a reasonable starting torque and hence two external admittances are used, one for the starting period and another for normal running. The phase convertor used for the starting period could be designed for the production of maximum starting torque, but since the convertor admittance for this condition is fairly large and is associated with large currents, designers favour the use of a convertor for optimum starting torque per ampere or starting torque per ampere squared. Since large unbalance results in wasteful heating, vibration and noise, phase convertors are sometimes chosen to produce minimum unbalance even at start. Whatever criterion is used for the design of the phase convertor, it is essential that the selected convertor should give a satisfactory run-up performance and not be associated either with high current densities in the auxiliary winding or with a high voltage across its terminals.

The obvious datum for any general comparison is the performance under balanced 2-phase operation, and in the following Sections, all performance equations are discussed on this basis.

(2.2) Starting Performance

At standstill $Y_1 = Y_2 = Y_s$, where Y_s is the normal standstill input admittance of the main winding. Substituting $Y_1 = Y_2 = Y_s$ in eqns. (7), (8), (9), (10) and (11), the performance equations at standstill are as follows:

$$V_1 = \frac{V[Y_s + k(k-j)Y]}{2(Y_s + k^2Y)} \quad (12)$$

$$V_2 = \frac{V[Y_s + k(k+j)Y]}{2(Y_s + k^2Y)} \quad (13)$$

$$I_m = VY_s \quad (14)$$

$$I_a = \frac{VY_sY}{Y_s + k^2Y} \quad (15)$$

and

$$I = \frac{[Y_s^2 + (1+k^2)YY_s]V}{Y_s + k^2Y} \quad (16)$$

(2.2.1) Starting Torque.

The ratio of starting torque, T, to the starting torque under balanced 2- phase operation, T_b, is given by the expression

$$\frac{T}{T_b} = \frac{|V_1|^2 - |V_2|^2}{|V^2|} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

which, from eqns. (12) and (13), reduces to

$$\frac{T}{T_b} = \frac{ky \sin \alpha}{k^4 + y^2 + 2k^2 y \cos \alpha} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where $y = |Y_s/Y|$, $\alpha = \beta - \phi$, and β and ϕ are the arguments of Y and Y_s, respectively.

The torque ratio given by eqn. (18) can be seen to be a function of the three dimensionless parameters, k, y and α, and hence is applicable to any induction motor with any form of phase convertor. For normal induction motors ϕ lies between $-20°$ and $-70°$, and for passive external elements β lies between the values of 90° for a pure capacitor and $-90°$ for a pure inductor, so that the practical limits for α are 160° and $-70°$. It may be stressed that, although theoretically windings can be designed to make $(Y_a - Y_m/k^2)$ have a negative conductance and hence make β have any value between 0 and 2π, in practice $(Y_a - Y_m/k^2)$ is either zero (symmetrical 2-phase winding) or has a small positive conductance. Hence, in what follows, α will be considered to vary within the limits $-70°$ and 160°.

Consideration of eqn. (18) shows that, for any given values of k and α, the torque ratio is a maximum when $y = k^2$, i.e. when the convertor impedance is k^2 times the standstill impedance of the main winding. With this value of y, eqn. (18) reduces to

$$\left(\frac{T}{T_b}\right)_{max} = \frac{\sin \alpha}{2k(1 + \cos \alpha)} \quad . \quad . \quad . \quad (18a)$$

Jordan and Lax[5] arrive at exactly the same equation for the maximum torque ratio, but then wrongly infer that this is a maximum when $\cos \alpha = 0$. It is easy to see that the ratio continues to increase as $\cos \alpha$ becomes more and more negative, finally becoming asymptotic when $\cos \alpha = -1$. In practice α for capacitor motors lies between 110° and 160°, and a much larger torque than that suggested by Jordan and Lax[5] is obtained.

It can easily be shown that, in general, when y and k are fixed, the torque ratio given by eqn. (18) is a maximum when $\alpha = \text{arc} \cos [-2k^2 y/(k^4 + y^2)]$, and when y and α are fixed for

$$k = \left[\frac{-y \cos \alpha \pm y\sqrt{(3 + \cos^2 \alpha)}}{3}\right]^{1/2}$$

Variations of T/T_b with α and y for some typical values of k are shown in Figs. 2A, 2B and 2C.

For discussion of the merits of various different types of starting, let a machine be considered in which $k = 1$. The maximum torque ratio is then given by

$$\left(\frac{T}{T_b}\right)_{max} = \frac{\sin \alpha}{2(1 + \cos \alpha)} \quad . \quad . \quad . \quad . \quad (18b)$$

and is shown plotted against α as curve (i) in Fig. 2A. Referring to this curve the performance of different types of phase convertor may be summarized as follows:

(a) *Capacitor and Resistor* ($160° > \alpha > 20°$): The maximum starting torque can have any value between approximately 9 and 280% of the balanced 2-phase torque. For pure capacitor starting, the range is 70–280%, the effect of resistance for all normal values of ϕ being to decrease the torque.

(b) *Pure Resistor* ($70° > \alpha > 20°$): The maximum starting torque lies within the range 9–35%. Most split-phase and resistance-start motors come in this category, and since $\beta = 0$, the torque depends

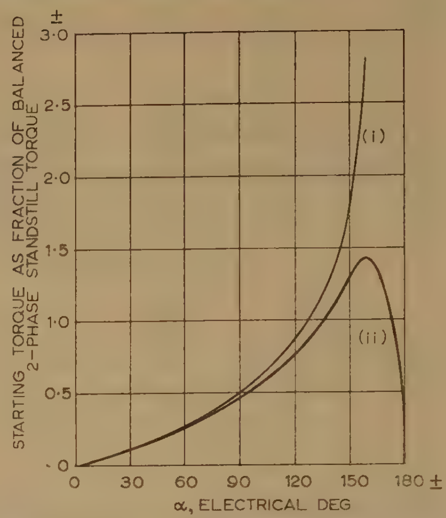

Fig. 2A.—Variation of starting torque with α.

(i) $k = 1 \cdot 0$, $y = 1 \cdot 0$
(ii) $k = 1 \cdot 0$, $y = \sqrt{2}$
Negative torques are associated with negative values of α.

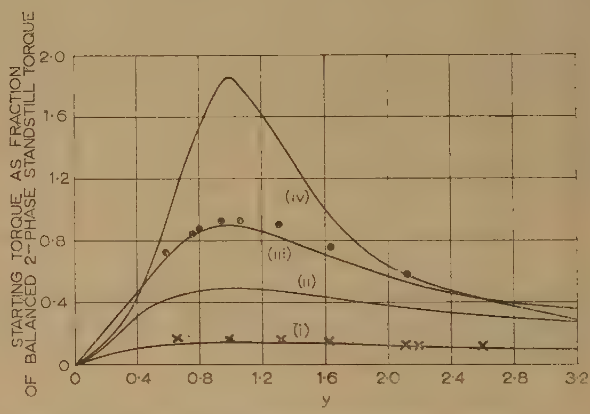

Fig. 2B.—Variation of starting torque with y when $k = 1 \cdot 0$.

(i) $\alpha = 31 \cdot 5°$
(ii) $\alpha = 90°$
(iii) $\alpha = 121 \cdot 5°$
(iv) $\alpha = 150°$
× Test points for $\alpha = 31 \cdot 5°$
● Test points for $\alpha = 121 \cdot 5°$

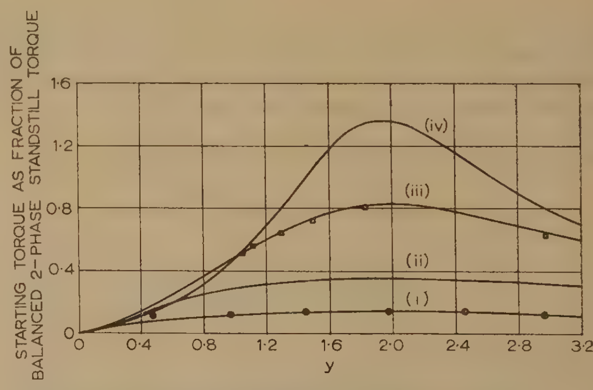

Fig. 2C.—Variation of starting torque with y when $k = 1 \cdot 4$.

(i) $\alpha = 43°$
(ii) $\alpha = 90°$
(iii) $\alpha = 133°$
(iv) $\alpha = 150°$
● Test points for $\alpha = 43°$
□ Test points for $\alpha = 133°$

entirely on ϕ and is extremely low for good power-factor operation (i.e. small values of ϕ).

(c) *Inductor and Resistor* ($70° > \alpha > -70°$): Again the starting torque cannot exceed 35%. It is negative for a pure inductor, and the effect of resistance is to make it decreasingly negative or increasingly positive.

Referring to Figs. 2A and 2B, it can be seen that the variation of starting torque with y is very small for small values of α and the choice of convertor admittance for maximum starting torque becomes significant only when α exceeds 90°. It is evident that the best type of convertor from the torque point of view is a pure capacitor. When properly selected it can give a torque greater than the balanced 2-phase value.

So far the effect of the variation of k on the torque ratio has been neglected. It can be seen from eqn. (18*a*) that the maximum torque ratio is inversely proportional to k, and hence by selecting a small value of k, a large torque could be obtained. However, this would be at the expense of large capacitor sizes ($y = k^2$) in capacitor-start motors and large currents in the auxiliary winding (since this current, even under balanced operation, is $1/k$ times the main winding current). In capacitor-start-and-run motors, k is usually fixed by design considerations for near balanced operation at load (see Section 2.2.2) and cannot be treated as a variable for meeting the starting-torque requirement of the machine.

(2.2.2) **Unbalance Factor.**

The operation of a 2-phase motor from a single-phase supply is inherently an unbalanced operation of the motor and hence is associated with all the drawbacks of such operation, i.e. heating, noise, vibration, etc. The degree of unbalance present under any condition of operation can be conveniently expressed by an unbalance factor U, defined as the ratio of the negative-sequence to the positive-sequence current, $|I_2/I_1|$, which is equal to $|V_2Y_2/V_1Y_1|$. Hence, from eqns. (7) and (8), the general equation for the unbalance factor is given by

$$U = \left|\frac{Y_2[Y_1 + k(k+j)Y]}{Y_1[Y_2 + k(k-j)Y]}\right| \quad . \quad . \quad . \quad (19)$$

From examination of the above equation the condition for $U = 0$, i.e. for perfectly balanced two-phase operation of the machine, is that $Y_1 + k(k+j)Y = 0$, which is satisfied only if

$$\left.\begin{aligned} k &= -\cot\alpha_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a) \\ \text{and } |Y_1/Y| &= -\cot\alpha_1 \operatorname{cosec}\alpha_1 = k\sqrt{(1+k^2)} \quad . \quad (b) \end{aligned}\right\} \quad (20)$$

where $\alpha_1 = \beta - \phi_1$, ϕ_1 being the argument of Y_1.

Since both k and $|Y_1/Y|$ must be positive, α_1 must be in the second quadrant to satisfy eqns. (20). Another possible condition for perfectly balanced operation is when $U = \infty$, i.e. $I_1 = 0$, and the machine then operates with reversed phase sequence. Owing to this reversal of the phase sequence Y_1 and Y_2 in eqn. (19) get interchanged, and the condition for $I_1 = 0$ is that

$$Y_1 + k(k-j)Y = 0$$

which is satisfied if

$$\left.\begin{aligned} k &= \cot\alpha_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a) \\ \text{and } |Y_1/Y| &= -\cot\alpha_1 \operatorname{cosec}\alpha_1 = k\sqrt{(1+k^2)} \quad (b) \end{aligned}\right\} \quad (21)$$

To satisfy eqns. (21), α_1 should be in the third quadrant. Since, in normal design and operation of single-phase machines, α_1 has the limits of 160° and −70°, eqns. (21) are never satisfied in practice.

It is obvious that the only type of convertor that can possibly give balanced 2-phase operation is a pure capacitor or a capacitor-resistor combination. In capacitor motors, as stated earlier, the auxiliary winding and the running capacitor are designed to give balanced 2-phase operation under normal load. Since $\alpha_1 = \beta - \phi_1$ and $\beta \simeq 90°$ in such motors, k for balanced operation must equal $-\tan\phi_1$. Capacitor motors usually operate under load at a power factor between 0·6 and 0·85, and the limiting values for k are therefore 1·35 and 0·6. Once the machine has been designed it can give balanced operation at only one power factor determined by $\phi_1 = \arctan(-k)$.

In normal induction motors both the positive-sequence impedance and power factor vary continuously as the motor is started from rest and runs up to take the load. It is therefore impossible to satisfy eqns. (20) for more than one point of the torque/speed curve unless k has more than one value (i.e. by means of a tapped auxiliary winding) and variable convertor admittances are available. For this reason, it is usual to expect unbalance to be present during the starting period of most single-phase motors, and the best the designer can do is to try and make it as small as possible. On substituting $Y_1 = Y_2 = Y_s$ in eqn. (19) and simplifying, the unbalance factor at standstill is given as

$$U = \left[\frac{y^2 + k^2(1+k^2) + 2ky(k\cos\alpha - \sin\alpha)}{y^2 + k^2(1+k^2) + 2ky(k\cos\alpha + \sin\alpha)}\right]^{1/2} \quad (22)$$

Examination of eqn. (22) shows that, for given values of k and α, the unbalance factor is a minimum when $y = k\sqrt{(1+k^2)}$, for given values of k and y when $\alpha = \arccos\{-2k^2y/[y^2 + k^2(1+k^2)]\}$, and for given values of y and α when

$$k = \left\{\frac{-(1+2y\cos\alpha) \pm \sqrt{[(1+2y\cos\alpha)^2 + 12y^2]}}{6}\right\}^{1/2}$$

Calculated values of U for a range of values of α, y and k are shown in Figs. 3A, 3B and 3C. Since for negative values of

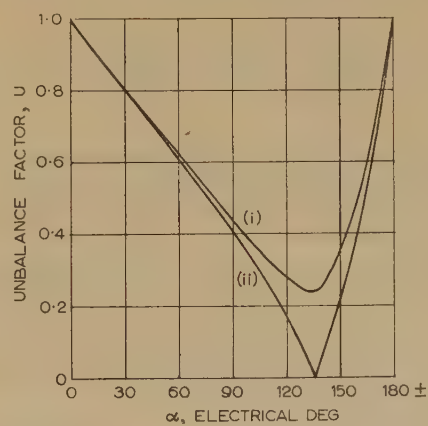

Fig. 3A.—Variation of unbalance factor with α.
(i) $k = 1\cdot0, y = 1\cdot0$
(ii) $k = 1\cdot0, y = \sqrt{2}$

α (i.e. α in the third or fourth quadrant) the starting torque is negative [eqn. (18)] and the motor runs up with reversed phase sequence, the unbalance factor in Fig. 3A for negative values of α has been redefined as the ratio $|I_1/I_2|$. This factor can then be seen to have the same numerical value as for corresponding positive values of α.

It has already been shown that only a capacitor-resistor type of phase convertor can, under certain conditions, give balanced 2-phase operation. It is, however, instructive to compare the degree of unbalance at starting obtained by using different types of convertor, and, for this purpose, a machine with $k = 1$ is con-

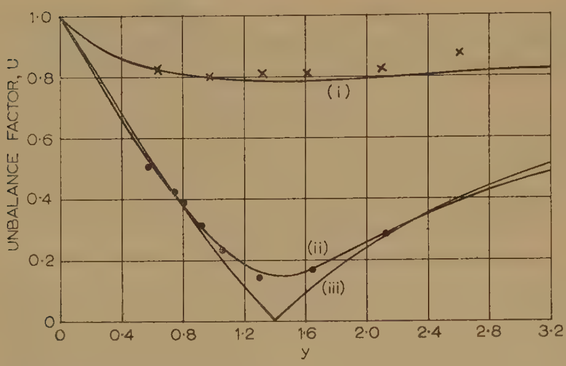

Fig. 3B.—Variation of unbalance factor with y when $k = 1 \cdot 0$.

(i) $\alpha = 31 \cdot 5°$
(ii) $\alpha = 121 \cdot 5°$
(iii) $\alpha = \text{arc cot}(-k) = 135°$
× Test points for $\alpha = 31 \cdot 5°$
● Test points for $\alpha = 121 \cdot 5$

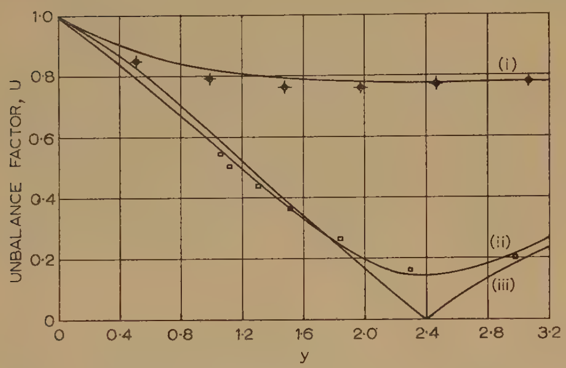

Fig. 3C.—Variation of unbalance factor with y when $k = 1 \cdot 4$.

(i) $\alpha = 43°$
(ii) $\alpha = 133°$
(iii) $\alpha = \text{arc cot}(-k) = 145 \cdot 5°$
● Test points for $\alpha = 43°$
□ Test points for $\alpha = 133°$

sidered. From curve (ii) in Fig. 3A, which represents the minimum unbalance condition for this machine ($y = k\sqrt{(1 + k^2)} = \sqrt{2}$), the performance of different phase convertors can be summarized as follows:

(*a*) *Capacitor and Resistor* ($160° > \alpha > 20°$): The unbalance factor U has a minimum value between zero and approximately 86%, zero being obtained at $\alpha = 135°$, i.e. arc cot $(-k)$. If numerically $\phi > 45°$ a capacitor-resistor combination is required to give perfect balance; if $\phi = 45°$ a pure capacitor will suffice, and, if $\phi < 45°$, perfect balance is not possible and minimum unbalance is obtained with a pure capacitor giving a value of $y = \sqrt{2}$.

(*b*) *Pure Resistor* ($70° > \alpha > 20°$): U has a minimum value between 54 and 86%. All split-phase and resistance-start motors therefore operate with a very high degree of unbalance.

(*c*) *Inductor and Resistor* ($70° > \alpha > -70°$): The minimum unbalance U lies between 54 and 100%. A high degree of unbalance is therefore present in all reactance-start motors.

It is thus evident that unbalance cannot be kept very low in any single-phase operation of an asymmetrical 2-phase motor, except when it is used as a capacitor motor. This is the main reason for the widespread use of capacitor motors for most single-phase applications except in the very small fractional-horse-power range.

(2.2.3) Starting Currents.

The currents in the main and the auxiliary winding, and the current drawn from the supply at standstill, are given by eqns. (14), (15) and (16), respectively. I_m is directly proportional to the standstill input admittance Y_s, and is independent of the convertor admittance Y. The current in the auxiliary winding can be rewritten in the form

$$|I_a| = \frac{V|Y_s|}{(k^4 + 2k^2 y \cos\alpha + y^2)^{1/2}} \quad . \quad . \quad (23)$$

Since I_{ab}, the convertor current under balanced 2-phase operation is VY_s/k, the ratio of the auxiliary current under single-phase operation to that under balanced 2-phase operation is

$$\left|\frac{I_a}{I_{ab}}\right| = \frac{k}{(k^4 + 2k^2 y \cos\alpha + y^2)^{1/2}} \quad . \quad . \quad . \quad (24)$$

Examination of eqn. (24) shows that the current $|I_a|$ is $1/k$ times $|I_{ab}|$ when $y = 0$ and is zero when $y = \infty$, the former being a pure single-phase operation with the two windings in parallel and the latter with the auxiliary winding open-circuited. For all positive values of cos α, i.e. for split-phase resistance- or reactance-start motors, $|I_a|$ is less than $1/k$ times $|I_{ab}|$. When a capacitive convertor is used, making cos α negative, it can easily be shown that $|I_a| = |I_{ab}|$ for either of the two values of y given by

$$y = -k^2 \cos\alpha \pm k\sqrt{(1 - k^2 \sin^2\alpha)} \quad . \quad . \quad . \quad (25)$$

and is greater than $|I_{ab}|$ when y lies between these two values, with a maximum at $y = -k^2 \cos\alpha$. Variations of $|I_a/I_{ab}|$ with y for different values of k and α are shown in Fig. 4.

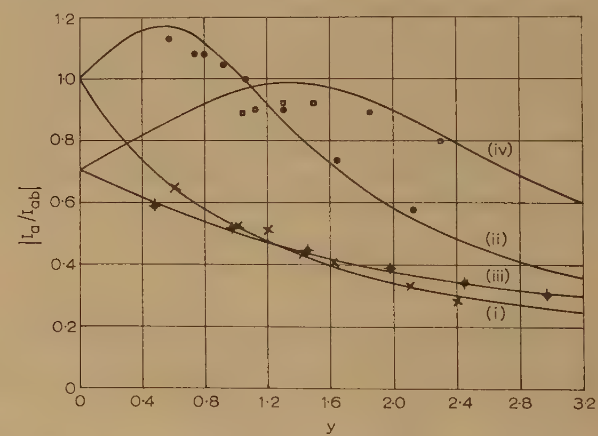

Fig. 4.—Variation of current in auxiliary winding with y.

(i) $k = 1 \cdot 0$, $\alpha = 31 \cdot 5°$
× Test points.
(ii) $k = 1 \cdot 0$, $\alpha = 121 \cdot 5°$
● Test points.
(iii) $k = 1 \cdot 4$, $\alpha = 43°$
⊕ Test points.
(iv) $k = 1 \cdot 4$, $\alpha = 133°$
□ Test points.

The other important current from the designer's viewpoint is the supply current, since it determines the total power taken from the supply. The ratio of the supply current under single-phase operation to that under balanced 2-phase operation is given by

$$\left|\frac{I}{I_b}\right| = \frac{k}{\sqrt{(1 + k^2)}}\left[\frac{y^2 + (1 + k^2)^2 + 2y(1 + k^2)\cos\alpha}{k^4 + 2k^2 y \cos\alpha + y^2}\right]^{1/2} \quad . \quad . \quad . \quad . \quad (26)$$

It can be shown from eqn. (26) that this ratio varies between $\sqrt{(1 + k^2)}/k$ for $y = 0$ and $k/\sqrt{(1 + k^2)}$ for $y = \infty$ and is equal to unity at $y = k\sqrt{(1 + k^2)}$. The latter condition is the same as that for minimum unbalance. Hence, for the minimum unbalance setting of y, the supply current at starting is always

equal to the balanced 2-phase standstill current irrespective of the value of α used. For all values of y between 0 and $k\sqrt{(1+k^2)}$, $|I|$ is greater than $|I_b|$, and for those between $k\sqrt{(1+k^2)}$ and ∞, $|I|$ is less than $|I_b|$.

Variations of $|I/I_b|$ with y for different values of k and α are shown in Fig. 5.

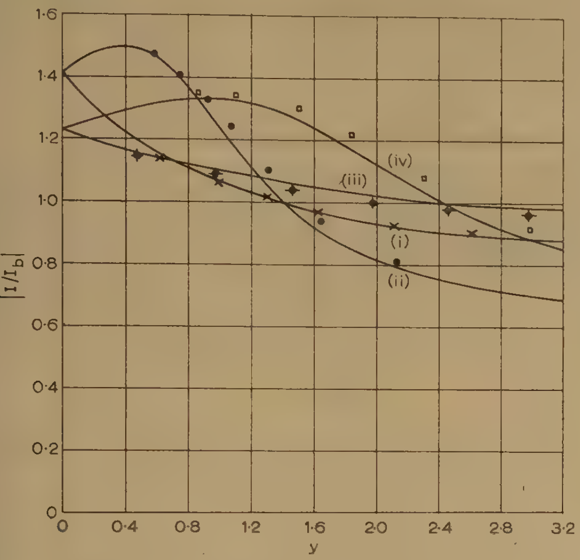

Fig. 5.—Variation of supply current with y.

(i) $k = 1{\cdot}0$, $\alpha = 31{\cdot}5°$
× Test points.
(ii) $k = 1{\cdot}0$, $\alpha = 121{\cdot}5°$
● Test points.
(iii) $k = 1{\cdot}4$, $\alpha = 43°$
⊕ Test points.
(iv) $k = 1{\cdot}4$, $\alpha = 133°$
□ Test points.

(2.2.4) Measures of Starting Quality.

The various starting characteristics of the machine, e.g. starting torque, unbalance, and starting currents, can be separately derived from results obtained in the previous Sections. However, it has been shown that conditions for optimum operation are widely different for the different characteristics. To permit the comparison of starting performances of different types of motor and to facilitate the most economic design of any given type, it is essential to evolve a starting criterion of the motor which is dependent on all the three characteristics, i.e. it should be a function of torque, unbalance and current. Since any criterion which is a function of current automatically includes the effect of unbalance, accepted measures of the so-called 'starting quality' are starting torque per ampere of supply current, and starting torque per ampere squared of supply current.

The ratio of starting torque per ampere of supply current under single-phase operation to that under balanced 2-phase operation is given, from eqns. (18) and (26), by

$$Q_1 = \left|\frac{T/I}{T_b/I_b}\right|$$
$$= \frac{y\sqrt{(1+k^2)}\sin\alpha}{(k^4+y^2+2k^2y\cos\alpha)^{1/2}[y^2+(1+k^2)^2+2y(1+k^2)\cos\alpha]^{1/2}} \quad . \quad . \quad . \quad . \quad (27)$$

The variation of Q_1 with y is of considerable interest, since some designers prefer to design the convertor to give a maximum value of Q_1. It can be shown from eqn. (27) that the curve of Q_1 against y has $dQ_1/dy = 0$ at

$$y = \tfrac{1}{2}\{-(1+2k^2)\cos\alpha \pm \sqrt{[1-(1+2k^2)^2\sin^2\alpha]}\}$$
$$\text{and } y = k\sqrt{(1+k^2)}$$

When cosec α is greater than $(1+2k^2)$, all solutions are real and the shape of the Q_1 curve is as shown by curve (a) in Fig. 6, the peaks occur at

$$y = \tfrac{1}{2}\{-(1+2k^2)\cos\alpha \pm \sqrt{[1-(1+2k^2)^2\sin^2\alpha]}\}$$

and the lowest point of the trough at $y = k\sqrt{(1+k^2)}$. In normal commercial capacitor-start motors α lies between 110°

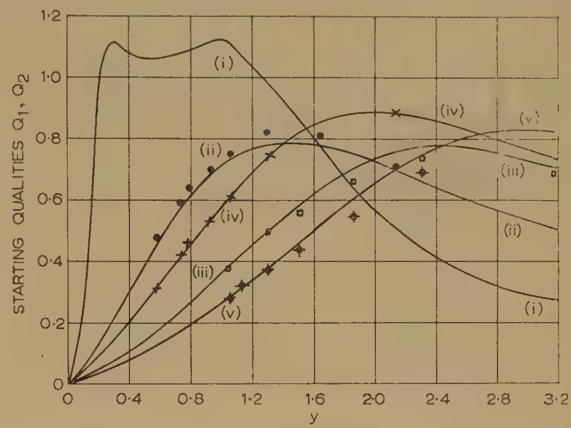

Fig. 6.—Variation of starting qualities, Q_1 and Q_2, with y.

(i) Q_1 when $k = 0{\cdot}5$, $\alpha = 150°$
(ii) Q_1 when $k = 1{\cdot}0$, $\alpha = 121{\cdot}5°$
● Test points.
(iii) Q_1 when $k = 1{\cdot}4$, $\alpha = 133°$
□ Test points.
(iv) Q_2 when $k = 1{\cdot}0$, $\alpha = 121{\cdot}5°$
× Test points.
(v) Q_2 when $k = 1{\cdot}4$, $\alpha = 133°$
⊕ Test points.

and 160°. To make cosec α greater than $(1+2k^2)$ will therefore limit k to a value less than $0{\cdot}98$. Usually k is greater than unity, in which case cosec $\alpha < (1+2k^2)$ and the only real solution for $dQ_1/dy = 0$ is given by $y = k\sqrt{(1+k^2)}$. d^2Q_1/dy^2 is then negative, and this condition gives the maximum starting torque per ampere for most practical motors. It is interesting that the condition is the same as that for minimum unbalance. Curves (ii) and (iii) in Fig. 6 shows the variation of Q_1 with y for two typical values of k and α.

When k and y are fixed, the value of α to give a maximum value of Q_1 is given from eqn. (27) by

$$\alpha = \text{arc cos}\,[-(y^2+k^2+k^4)/y(1+2k^2)]$$
$$\text{or } \alpha = \text{arc cos}\,[-y(1+2k^2)/(y^2+k^2+k^4)]$$

whichever is real. If y is selected to satisfy $y = k\sqrt{(1+k^2)}$, the best value of α for optimum Q_1 is

$$\alpha = \text{arc cos}\,[-2k\sqrt{(1+k^2)}/(1+2k^2)] \text{ or}$$
$$\text{arc cos}\,[-(1+2k^2)/2k\sqrt{(1+k^2)}]$$

whichever is real.

Starting torque per ampere squared of supply current is used as another criterion for the starting quality of single-phase induction motors. If Q_2 represents the ratio of this quantity under single-phase operation to that under balanced 2-phase operation,

$$Q_2 = \frac{T/|I|^2}{T_b/|I_b|^2} = \frac{(1+k^2)y\sin\alpha}{k[y^2+(1+k^2)^2+2y(1+k^2)\cos\alpha]} \quad (28)$$

Examination of the above equation shows that for given values

of k and y, Q_2 is a maximum when $\alpha = \arccos\{-2y(1+k^2)/[y^2 + (1+k^2)^2]\}$ and for given values of k and α, when $y = (1 + k^2)$. Variations of Q_2 with y for two typical values of k and α are shown by curves (iv) and (v) of Fig. 6.

The two starting-quality criteria so far discussed are based on two entirely different design outlooks. Some designers consider that for a given design the best use of materials is to obtain the maximum starting torque without exceeding a given current density in the windings, and hence they prefer use of 'starting-torque per ampere' as a basis for comparing performances of different single-phase machines. Advocates of 'starting torque per ampere squared' on the other hand, base their arguments on the fact that it is not the current but the square of the current which usually determines both the torque and the rating of the machine. When applying either of these criteria the base current is usually taken as the supply current. However, this ignores the separate effects of the currents in the two windings. Either or both the individual currents could, under certain conditions, be much greater than the supply current and hence neither criterion is very satisfactory.

In an earlier paper[6] the use of an alternative criterion, namely 'the starting torque per watt of stator copper loss', has been advocated as this includes the effect of all the individual winding currents. If Q_3 is the ratio of starting torque per watt of stator copper loss under single-phase operation to that under balanced 2-phase operation,

$$Q_3 = \frac{T/[|I_m|^2 r_m + |I_a|^2 r_m k^2]}{T_b/2|I_m|^2 r_m} = \frac{T}{T_b} \times \frac{2}{1 + k\left|\dfrac{I_a}{I_{ab}}\right|^2}$$

which, from eqns. (18) and (24), reduces to

$$Q_3 = \frac{2ky \sin\alpha}{k^4 + k^2 + 2k^2 y \cos\alpha + y^2} \quad . \quad . \quad . \quad (29)$$

Variations of Q_3 with y for two typical values of k and α are plotted in Fig. 7.

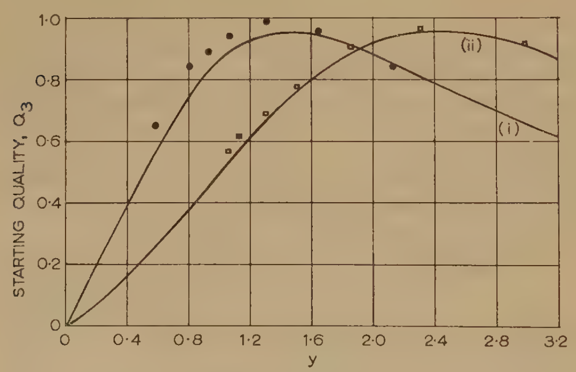

Fig. 7.—Variation of starting torque per watt of stator copper loss with y.

(i) $k = 1 \cdot 0$, $\alpha = 121 \cdot 5°$
● Test points.
(ii) $k = 1 \cdot 4$, $\alpha = 133°$
□ Test points.

It can be shown from eqn. (29) that, for given values of k and α, Q_3 is a maximum when $y = k\sqrt{(1 + k^2)}$; for given values of k and y when $\alpha = \arccos[-2k^2 y/(k^4 + k^2 + y^2)]$; and for given values of α and y when

$$k = \left\{\frac{-(1 + 2y\cos\alpha) \pm \sqrt{[(1 + 2y\cos\alpha)^2 + 12y^2]}}{6}\right\}^{1/2}$$

These conditions are identical to those established in Section 2.2.2 for operation of the machine with minimum unbalance, and the value of y is the same as for the condition of maximum torque per supply ampere. In the opinion of the authors, the starting quality defined by Q_3 is more satisfactory than those defined by either Q_1 or Q_2 and a machine operating under optimum conditions of Q_3 utilizes the materials in the most economic way. Under this condition, since the machine is starting with minimum unbalance, noise and vibration are also at their lowest level.

(2.2.5) Voltage across the Phase Convertor.

The voltage developed across the external phase convertor is another quantity which the designer must take into account to enable him to fix the voltage rating of the convertor and also to ensure safe operation of the machine. This voltage is given directly by the quotient I_a/Y_{ext}. In most motors in which external phase convertors are used, $(Y_a - Y_m/k^2)$ is usually small compared with Y_{ext}. Hence the voltage across the external phase convertor can be considered as approximately equal to I_a/Y. The ratio of this voltage to the supply voltage is then given by

$$\left|\frac{V_Y}{V}\right| = \left|\frac{I_a/Y}{V}\right| = \frac{y}{(k^4 + 2k^2 y\cos\alpha + y^2)^{1/2}} \quad . \quad (30)$$

It can easily be shown that the ratio is a maximum when $y = -k^2/\cos\alpha$. The condition is reached only in capacitor-start motors, since $\cos\alpha$ has to be negative to make y real. Substituting this value of y in eqn. (30), the maximum voltage across the convertor, expressed as a fraction of the supply voltage, is given by $\operatorname{cosec}\alpha$, and is therefore independent of the value of k. Since in the normal range of motor operation, i.e. when $110° < \alpha < 160°$, $\operatorname{cosec}\alpha$ varies from $1 \cdot 06$ to $2 \cdot 92$, a very

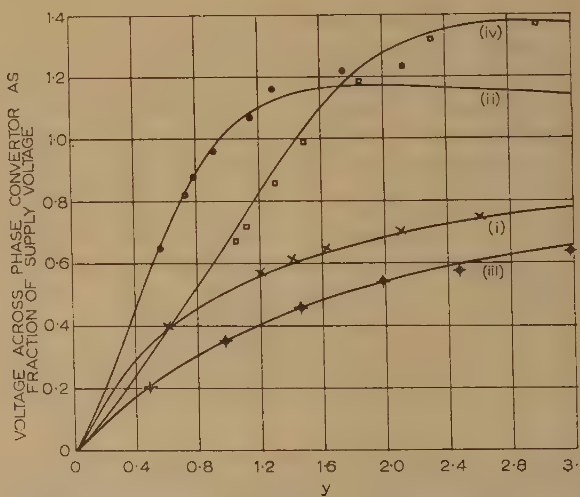

Fig. 8.—Variation of voltage across convertor with y.

(i) $k = 1 \cdot 0$, $\alpha = 31 \cdot 5°$
× Test points.
(ii) $k = 1 \cdot 0$, $\alpha = 121 \cdot 5°$
● Test points.
(iii) $k = 1 \cdot 4$, $\alpha = 43°$
⊕ Test points.
(iv) $k = 1 \cdot 4$, $\alpha = 133°$
□ Test points.

high voltage may be set up across the capacitor unless proper care is taken in design and selection of α, y and k. Variation of $|V_Y/V|$ with y for a few typical values of α and k are shown in Fig. 8.

(2.3) Run-up Performance

Although the theoretical calculation of run-up performance of asymmetrical 2-phase machines supplied from a single-phase source is beyond the scope of the paper, it is nevertheless essential to establish that the convertor selected for starting must give satisfactory run-up characteristics. Ensuring this usually requires complete calculation of the torque/speed curve in the run-up period from the equivalent circuits for positive- and negative-sequence operation. This will necessarily involve considerable work unless computers are available. Furthermore, the effects of harmonics and stray load losses make the calculated values of torque differ considerably from those actually developed by the machine.

It is usual to design a phase convertor to meet the starting requirements and then to verify experimentally that the run-up performance is satisfactory. It is therefore sufficient for the purpose of the paper to deduce some general conclusions from direct experiments on a standard machine.

(3) EXPERIMENTAL WORK

(3.1) General

Tests were carried out on a 4-pole 50 c/s squirrel-cage induction motor having the following nameplate ratings: single phase, 3 h.p., 240 volts, 16·4 amp, 1 440 r.p.m. The machine was standard in all respects except that the auxiliary winding was split into three separate windings with the respective number of turns equal to 1·2, 0·1 and 0·1 times that of the main winding. By using the 1·2 winding alone, or in series aiding or in series opposition with one or both of the 0·1 windings, any of the five different turns ratios 1·0, 1·1, 1·2, 1·3 and 1·4 could be obtained on the same machine. For test purposes the motor was coupled to a d.c. dynamometer whose speed could be controlled by means of a Ward Leonard arrangement.

Results for balanced 2-phase operation, which were used as data for all single-phase performance comparisons, were obtained by supplying the motor windings with voltages proportional in magnitude to their effective number of turns and differing in time phase by 90°, and by making $Y_a - Y_m/k^2$ as near zero as possible through the insertion of a suitable resistance in series with the main winding. This external resistance, whose magnitude was different for different values of k, was left in circuit throughout the tests and was considered as part of the main winding. In single-phase tests, therefore, the phase-convertor admittance Y became equal to the admittance of the external convertor Y_{ext} since $Y = Y_a - Y_m/k^2 + Y_{ext}$. Two types of external convertors were used for tests—a variable resistor ($\beta = 0°$) and a variable capacitor bank ($\beta = 90°$).

(3.2) Starting Performance

Since the starting current in most induction motors is 4–6 times the full-load current, prolonged standstill tests can be carried out only at a fraction of the rated voltage. The supply voltages for the 2-phase operation of the test motor were maintained at 70 volts for the main winding and 70k volts for the auxiliary winding, while all single-phase measurements were obtained at a supply voltage of 70 volts. Use of this low voltage (about 29% of the rated value) also ensured that the effects of saturation on the magnitude of winding impedances were negligible. The standstill input admittance, Y_s, of the main winding under these conditions was $0{\cdot}13\angle{-31{\cdot}5°}$ when $k = 1$ and $0{\cdot}183\angle{-43°}$ when $k = 1{\cdot}4$. Y_s differs both in magnitude and phase for the two values of k because different external resistances in series with the main winding were required to satisfy the condition $Y_a - Y_m/k^2 \simeq 0$.

To ensure that the position of the rotor does not affect performance measurements at standstill, the starting characteristics for both 2-phase and single-phase operations were obtained by extrapolation of graphs of readings taken over a range of very low speeds in both directions, i.e. at slip values slightly below and above unity. For the measurement of unbalance factor, the bridge circuits described in a recent paper[7] for the direct measurement of symmetrical-component quantities in 2-phase systems were used with slight modifications to suit the case of machines having turns ratios other than unity.

The test results are recorded on the same Figures as the corresponding theoretical results (i.e. on Figs. 2B, 2C, 3B, 3C, 4, 5, 6, 7 and 8), and as reference to these Figures shows, there is sufficiently close agreement between theory and practice to provide complete confirmation of the theory given in Section 2. Though the machine was tested for all five values of k, i.e. $k = 1{\cdot}0$, $1{\cdot}1$, $1{\cdot}2$, $1{\cdot}3$ and $1{\cdot}4$, experimental results are given only for $k = 1{\cdot}0$ and $1{\cdot}4$. The agreement between calculated and measured characteristics for $k = 1{\cdot}1$, $1{\cdot}2$ and $1{\cdot}3$, however, is of the same order as that for $k = 1{\cdot}0$ and $1{\cdot}4$.

(3.3) Run-up Performance

The run-up performance was investigated on lines exactly similar to those used in the study of single-phase operation of 3-phase induction motors.[6] Two series of tests were carried out, the first being with the object of studying the variation of torque with y at different speeds. Results of these tests for the machine with $k = 1{\cdot}4$ are shown in Fig. 9A as torque versus y, and in Fig. 9B in the more familiar form of torque/speed curves.

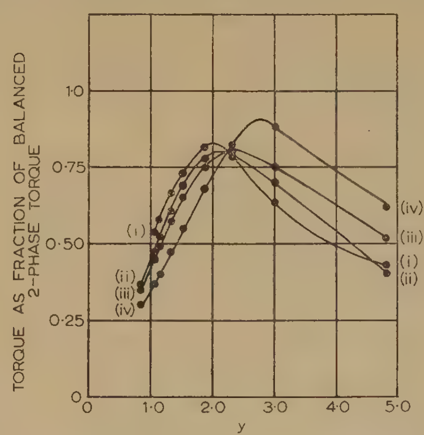

Fig. 9A.—Variation of torque with y at different speeds measured on the test machine when $k = 1{\cdot}4$.

(i) Standstill.
(ii) 400 r.p.m.
(iii) 800 r.p.m.
(iv) 1 200 r.p.m.

In the second series, variation of run-up times with y under different loading conditions were studied, the run-up time being defined as the time taken by the machine to run up from standstill to two-thirds of synchronous speed. Results of the test relating to the machine with $k = 1{\cdot}4$ are shown in Fig. 10.

(3.3.1) Discussion of Results.

The results shown in Figs. 9A, 9B and 10 are, as expected, very similar to those obtained for the run-up performance of a 3-phase induction motor connected to a single-phase supply.[6] The value of y to give maximum torque increases with increase in speed, and hence a continuously-variable convertor admittance would be required if maximum possible torque were desired at all

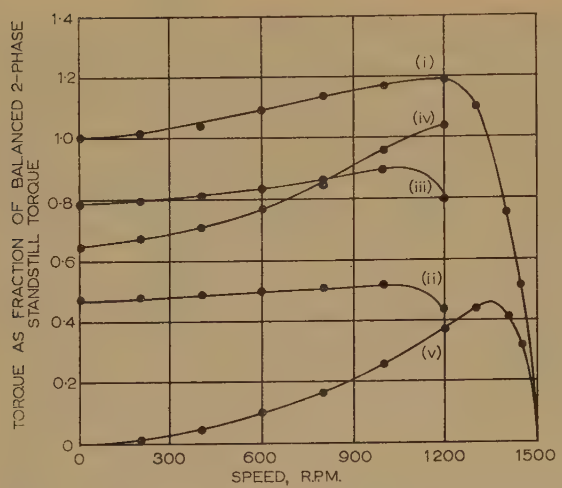

Fig. 9B.—Variation of torque with speed for different values of y, measured on the test machine when $k = 1 \cdot 4$.

(i) Balanced 2-phase operation.
(ii) Single-phase operation with capacitor $y = 1 \cdot 0$.
(iii) Single-phase operation with capacitor $y = 1 \cdot 9$.
(iv) Single-phase operation with capacitor $y = 3 \cdot 0$.
(v) Single-phase operation, auxiliary on open-circuit $y = \infty$.

speeds. In practice, it is usually sufficient to design a convertor which gives a reasonable torque, say about 70–80% of the balanced 2-phase value, at all speeds during the run-up period, thus ensuring a satisfactory run-up performance.

It is interesting to note from Fig. 10 that the run-up time under all the three loading conditions is a minimum when

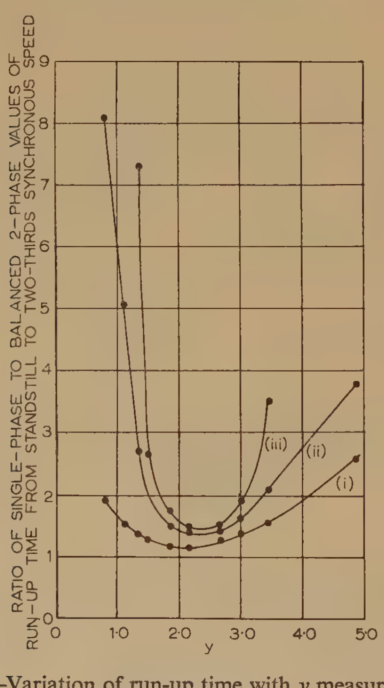

Fig. 10.—Variation of run-up time with y measured on the test machine when $k = 1 \cdot 4$.

(i) Motor on no load.
(ii) and (iii) Motor driving a separately-excited generator with different resistances across the armature.

$y \simeq 2 \cdot 4$, which, for the value of k used (i.e. $1 \cdot 4$), is the condition for minimum unbalance at starting. The same result was obtained when the motor was tested with other values of k; i.e. in all cases the run-up time was a minimum when the convertor admittance satisfied the condition for minimum unbalance at starting $[y = k\sqrt{(1 + k^2)}]$. Since the run-up time is a function of the speed/torque curve, the shape of which varies widely from machine to machine, the authors do not feel justified in claiming that this result is representative of all motors of this type. The evidence of the tests, however, confirms that, when the convertor is designed for optimum starting performance (i.e. for minimum unbalance at starting), the run-up performance is very satisfactory.

The discussion in Section 2.3 is based on steady-state theory, and test results recorded in Figs. 9A and 9B apply to this condition. The dynamic characteristics usually differ from the steady-state characteristics because of speed-variation effects, but the results of a recent investigation[8] indicate that the difference is not very significant at relatively high values of slip. In any case, the run-up time ratios recorded in Fig. 10 include dynamic effects, and are therefore applicable to the normal starting operation.

(4) CONCLUSIONS

The method of expressing the starting performance of any polyphase induction motor connected to a single-phase supply system, in terms of dimensionless parameters, first demonstrated for the case of a 3-phase motor in an earlier paper,[6] has been extended to cover the case of asymmetrical 2-phase machines. In addition to the two parameters y and α already encountered in the 3-phase case, another parameter k, the turns ratio, is employed to take into consideration the asymmetry in the 2-phase stator windings. The performance equations, since they contain only dimensionless quantities, are applicable to machines of any rating and, with suitable interpretation of parameters, to most types of single-phase motors.

It is shown in the paper that the best starting performance is obtained with a capacitive convertor selected to give minimum unbalance, satisfying the equation $y = k\sqrt{(1 + k^2)}$. The actual choice of the size and type of the phase convertor, however, depends largely upon the particular application of the machine, and values of admittance other than that for optimum operation are in everyday use. For this reason, the curves are drawn to cover a wide range of variation of parameters. Performance characteristics for any combination outside this range can, however, be very easily calculated from the general equations.

The designer usually knows almost instinctively the performance he can obtain under balanced operation from a standard polyphase induction motor of any given frame size. It is hoped that use of the performance equations given in this paper and the earlier related one[6] will enable him to predetermine with ease the starting performance of the same machine under various types of single-phase operation, and thus help him in selecting the most suitable design to satisfy any desired characteristics.

(5) ACKNOWLEDGMENTS

The authors wish to express their appreciation of the facilities placed at their disposal in the Electrical Engineering Department of the University of Bristol. They are also indebted to Dr. J. E. Brown, Lecturer in the Department, for his helpful advice and criticism during the investigation and in the subsequent preparation of the paper, and to Mr. P. H. Dimambro, an honours student in the Department, for his co-operation in the practical tests.

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DISCUSSION ON

'PERFORMANCE CALCULATIONS OF THE CAPACITOR MOTOR USING THE TRANSFORMER ANALOGUE ANALYSER'*

Professor G. R. Slemon (*Canada: communicated*): When the equivalent circuit given in Fig. 3 of the paper was first presented for discussion,† it was pointed out that there is one aspect in which the cross-field approach is inferior to the revolving-field approach. The rotor current in a single-phase motor has two basic components with frequencies of $(1 - v)$ and $(1 + v)$ times line frequency respectively. Because of skin effect in the rotor bars the resistances encountered by these two current components may be markedly different. Since the revolving-field theory provides for this effect while the cross-field theory cannot, the former has been preferred. On the other hand, it is agreed that the cross-field theory is preferable when viewing the stator of the motor, since it corresponds to the physical arrangement of the windings and also provides for the inclusion of saturation effects.

In my reply to the discussion on my paper,† I suggested a compromise in which the cross-field approach is used in representing the stator system while a revolving-field approach is used for the rotor. The two sections of the equivalent circuit are then linked by two ideal non-reciprocal transformers which perform the necessary symmetrical-component conversion from the 2-phase quantities in the stator to the two counter-rotating sets of polyphase quantities in the rotor. I would be pleased to hear the authors' comments on this approach.

The authors have encountered considerable discrepancies between calculated and measured torques, part of which they attribute to third- and fifth-harmonic space waves in the motor. In this regard I would again refer to my paper,† in which an equivalent circuit similar to that of Fig. 3 has been extended to include the effects of these space harmonics.

While I agree with the expression for torque in eqn. (2), I believe that there is an error in eqn. (5). The operator j has been omitted in the voltage expressions of eqn. (4). If this is inserted and substitution in eqn. (2) is made, it will be found that the torque is the imaginary part rather than the real part of the bracketed quantity in eqn. (5).

The results which the authors have obtained by including the effects of saturation on the magnetizing reactances are most interesting. I would appreciate their comments on the reduction in accuracy which would occur if (*a*) the mutual effects on saturation between the two axes had been neglected, or (*b*) a single saturated value had been used for X_d and X_q.

Dr. M. K. E. Ismail and **Mr. B. Adkins** (*in reply*): We apologize for not having included Dr. Slemon's paper in the Bibliography. The present paper derives the circuit from the equations and thus follows the most direct method of obtaining an 'equivalent circuit'.

The two-axis theory is of universal application, and it can certainly be applied to a deep-bar squirrel-cage motor. For a multiple squirrel cage it is necessary to add two more equations to eqns. (1) for each additional coupled rotor circuit. An equivalent circuit can then be set up for use with a transformer analogue analyser. However, the system becomes complicated for a rotor with distributed currents.

An alternative approach can be made using Park's two-axis equations, for which the axes are fixed relative to the rotor. It is then possible to eliminate the unwanted rotor currents from the equations and represent the rotor circuits, however complicated, by a single operational impedance, the components of which vary with frequency. With an unbalanced primary, however, the phase-to-axis transformation introduces the same components as the revolving-field theory, and it is not easy to allow for saturation.

The concept of operational impedance is of particular value in the study of machines with solid or deep-bar rotors, e.g. turbo-alternators, and it needs to be developed further. In general, many of the concepts which originated in the study of the synchronous machine can be applied to the induction motor. However, when the axes are fixed relative to the primary, the elimination of the unwanted rotor currents is a difficult problem, which has not yet been solved.

The discrepancy between measured and calculated torque is due more to 'stray losses' than to air-gap harmonics. The quadrature-axis flux can sometimes be very high, and under such conditions the mutual saturation effect is important. We thank Dr. Slemon for pointing out the error in eqns. (4) and (5).

* Ismail, M. K. E., and Adkins, B.: Paper No. 2851 U, April, 1959 (see 106 A, p. 175).
† Slemon, G. R.: 'Equivalent Circuits for Single-Phase Motors', *Transactions of the American I.E.E.*, 1956, 75, Part III, p. 1335.

DISCUSSION ON
'A GENERALIZED LOCUS DIAGRAM FOR INDUCTION MOTORS'*

Dr. V. Ahmad (*communicated*): The authors assume that the law of variation of secondary resistance with frequency in deep-bar rotors, due to skin effect, is fairly accurately known, but perhaps rely on the experimental results for assessing the variation of leakage reactance with currents in the stator and rotor. It seems to me that the ${}_{pu}X/{}_{pu}I$ curves shown in Fig. 3 were obtained by performing short-circuit tests on a few machines of similar design and rating and by taking the mean of the leakage reactances deduced from these tests at various currents up to 6–7 times the rated value. If so, the method is available for existing machines only, but may possibly be used for machines of different ratings if essentially of similar design by the application of empirical laws based on the experimental results. Evidently, the results cannot be generalized, and I do not think that the authors mean anything like this.

It may be mentioned, however, that once the relevant ${}_{pu}X/{}_{pu}I$ curve for any machine is available, its performance can be calculated accurately using the appropriate values of the parameters in its equivalent circuit under any condition of operation. For determining the best approximation of the leakage reactance to be used for any load, calculations would generally have to be repeated, but this would be no deterrent, particularly now that the facilities of digital computers and other calculating machines are becoming increasingly available. It may be appreciated, therefore, that the real problem is how to predict the X/I curve for any machine from its design data as accurately as possible.

As regards the comparison between the calculated and measured values in Fig. 11, I feel that the agreement is quite good, and that the discrepancies may not be as great as the authors appear to think.

Some reduction in the output torque of an induction motor is always observed in the motoring region because of the retarding effects of the high-frequency stray load losses, and I should have expected a much larger discrepancy on this characteristic. These losses, however, do not affect the parameters of the equivalent circuit, and the good agreement on the current/slip characteristic shows that the authors' method is quite reliable. A notable discrepancy in the region of slips of 0·7–0·9 is most certainly due to the harmonic effects.

I do not know whether the authors have performed any tests for determining the practical current-locus diagram, or preferably the admittance-locus diagram, at voltages sufficient to give 4–5 times rated current at starting, in order to compare it with the graphically derived one. I think it would be quite interesting to do that; temperature effects sometimes tend to mask the other effects, but experiments may be performed after running the machine for some time on load when the winding temperatures have become practically constant.

Messrs. P. Lomb and **G. Ellesworth** (*in reply*): Dr. Ahmad's assumption that the ${}_{pu}X/{}_{pu}I$ curves in Fig. 3 were based on experimental results obtained from a limited number of machines of similar design is quite correct. The curves cannot therefore be used indiscriminately with machines of different design.

Attempts are being made to establish means of determining the shape of the ${}_{pu}X/{}_{pu}I$ curves from design data. At present all that can be said is that, for a given geometrical pattern of laminations, the shape of the curve is influenced mainly by the following factors:

(*a*) The relationship between the end-winding leakage reactance, which is practically independent of saturation, and the other (saturation dependent) parts of the leakage reactance.

(*b*) The number of slots per pole per phase (q).

As far as (*b*) is concerned, reference may be made to W. Schuisky's investigations [Schuisky: 'Induktion Maschinen' (Springer, Wien, 1957)], according to which the effect of saturation is stronger if q is small (e.g. 1, 2 or 3) and weaker if q is higher.

Bearing in mind these two factors, it is possible to some degree to predict the shape of the curves. In general, for low-output motors, q is small; therefore the curves for smaller motors have a tendency to be steep as compared with those for bigger machines. On the other hand, for the same output, q usually decreases with increasing number of poles, and therefore the curves become steeper as the number of poles increases. This tendency is also influenced by the fact that the total leakage reactance of a 2-pole motor includes more end-winding reactance. It may also be worth observing that the curve of ${}_{pu}X/{}_{pu}I$ continues to rise below ${}_{pu}I = 1$, and its rise is quite steep between ${}_{pu}I = \frac{1}{2}$ and ${}_{pu}I = 1$.

We agree with Dr. Ahmad's remark that, given the necessary data, induction motor problems may be solved using the equivalent circuit and that modern computing aids greatly reduce the tediousness of such a project. It is worth noting, however, that the graphical method presented in the paper gives much valuable insight into the physical processes within the machine, and also the effect of individual parameter changes, which could easily be lost sight of in the course of mechanization of the computing process.

In conclusion, we wish to confirm that load tests covering the entire speed range have been performed using a dynamometer test-bed. Performance curves of the type illustrated in Fig. 11 were consistently obtained after making allowance for the temperature changes in the stator windings.

* Lomb, P., and Ellesworth, G.: Paper No. 2696 U, October, 1958 (see 105 A, p. 517).

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Published by The Institution, Savoy Place, London, W.C.2. *Telephone:* COVent Garden 1871. *Telegrams:* 'Voltampere, Phone, London.'
Printed by Unwin Brothers Limited, Woking and London.